

Ex 1.16

$$a) \sqrt[3]{\sqrt{7}} = (7^{1/2})^{1/3} = 7^{\frac{1}{2} \cdot \frac{1}{3}} = 7^{\frac{1}{6}} = \underline{\sqrt[6]{7}}$$

$$b) \sqrt[3]{2^{18} \cdot 5^{12} \cdot 3^3} = \sqrt[3]{2^{18}} \sqrt[3]{5^{12}} \sqrt[3]{3^3} = 2^6 \cdot 5^4 \cdot 3 = 2^2 \cdot 2^4 \cdot 5^4 \cdot 3 \\ = 4 \cdot (2 \cdot 5)^4 \cdot 3 = 12 \cdot 10^4 = \underline{120'000}$$

$$c) \sqrt[4]{64} \cdot \sqrt[4]{4} = \sqrt[4]{2^6 \cdot 2^2} = \sqrt[4]{2^8} = 2^2 = \underline{4}$$

$$d) \sqrt[5]{3^{15}} = 3^3 = \underline{27}$$

$$e) \left(\sqrt[8]{\sqrt[4]{\sqrt{2}}} \right)^{128} = \left(\left(\left(2^{1/2} \right)^{1/4} \right)^{1/8} \right)^{128} = 2^{\frac{128}{2 \cdot 4 \cdot 8}} = 2^2 = \underline{4}$$

$$f) \sqrt{3\sqrt{3}} = (3 \cdot 3^{1/2})^{1/2} = (3^{1+1/2})^{1/2} = (3^{3/2})^{1/2} = 3^{3/4} = \sqrt[4]{3^3} = \underline{\sqrt[4]{27}}$$

$$g) \sqrt[3]{5\sqrt{5\sqrt{5}}} = (5(5 \cdot 5^{1/2})^{1/2})^{1/3} = (5(5^{3/2})^{1/2})^{1/3} = (5 \cdot 5^{3/4})^{1/3} \\ = (5^{1+\frac{3}{4}})^{1/3} = (5^{7/4})^{1/3} = 5^{7/12} = \underline{\sqrt[12]{5^7}}$$

$$h) \sqrt{2^3\sqrt{2}} = (2 \cdot 2^{1/3})^{1/2} = (2^{1+1/3})^{1/2} = (2^{4/3})^{1/2} = 2^{2/3} = \sqrt[3]{2^2} = \underline{\sqrt[3]{4}}$$

$$i) \sqrt[3]{3^3 \sqrt[3]{3^4 \sqrt[3]{3^6}}} = (3(3^4 \cdot 3^2)^{1/3})^{1/3} = (3(3^6)^{1/3})^{1/3} \\ = (3 \cdot 3^2)^{1/3} = (3^3)^{1/3} = \underline{3}$$

$$j) \sqrt[3]{2 \sqrt[6]{\frac{2^{14}}{\sqrt[3]{2^6}}}} = \left(2 \left(\frac{2^{14}}{2^2} \right)^{1/6} \right)^{1/3} = \left(2 (2^{12})^{1/6} \right)^{1/3} = (2 \cdot 2^2)^{1/3} = (2^3)^{1/3} = \underline{2}$$