

Ex 3.1

$$\vec{a} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$a) \quad \vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} - 3 \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1+6+3 \\ -1+8-6 \end{pmatrix} = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$$

$$\vec{w} = 4 \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \frac{3}{2} \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 - \frac{9}{2} + \frac{1}{2} \\ -4 - 6 - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -11 \end{pmatrix}$$

$$b) \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix} \stackrel{?}{=} k \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Leftrightarrow \begin{cases} 1 = -k \\ -1 = 2k \end{cases} \Leftrightarrow \begin{cases} k = -1 \\ k = -1/2 \end{cases} \neq \Rightarrow \text{non colinéaires}$$

$$\text{ou } \det(\vec{a}, \vec{c}) = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 1 \cdot 2 - (-1)(-1) = 1 \neq 0 \Rightarrow \text{non colinéaires.}$$

$$c) \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix} \stackrel{?}{=} k \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Leftrightarrow \begin{cases} 1 = 3k \\ -1 = 4k \end{cases} \Leftrightarrow \begin{cases} k = 1/3 \\ k = -1/4 \end{cases} \neq \Rightarrow \text{non colinéaires.}$$

$$d) \quad \det(\vec{b}, \vec{c}) = \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} = 3 \cdot 2 - 4 \cdot (-1) = 10 \neq 0 \Rightarrow \text{non colinéaires.}$$

$$e) \quad \vec{c} = x\vec{a} + y\vec{b} \Leftrightarrow \begin{pmatrix} -1 \\ 2 \end{pmatrix} = x \begin{pmatrix} 1 \\ -1 \end{pmatrix} + y \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Leftrightarrow \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} x+3y \\ -x+4y \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} x+3y = -1 \\ -x+4y = 2 \end{cases}$$

$$\underline{7y = 1}$$

$$\underline{y = \frac{1}{7}}$$

$$\Rightarrow x + 3 \cdot \frac{1}{7} = -1$$

$$x = -1 - \frac{3}{7}$$

$$\underline{\underline{x = -\frac{10}{7}}}$$

$$f) \quad \vec{e}_1 = m\vec{a} + p\vec{b} \Leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} = m \begin{pmatrix} 1 \\ -1 \end{pmatrix} + p \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} m+3p = 1 \\ -m+4p = 0 \end{cases}$$

$$\underline{7p = 1}$$

$$\underline{p = \frac{1}{7}}$$

$$\Rightarrow -m + \frac{4}{7} = 0$$

$$\underline{\underline{m = -\frac{4}{7}}}$$