# Résolution par changement de variable

$$a \cdot u^{2x} + b \cdot u^{x} + c = 0$$

Pour résoudre on effectue un changement de variable :

on pose $y = u^x \Rightarrow y^2 = u^{2x}$

$\Rightarrow \quad a \cdot y^2 + b \cdot y + c = 0$

**Exemple** : $5^{2x} - 6 \cdot 5^x + 5 = 0$

on pose $y = 5^x$

$\Rightarrow \quad y^2 - 6y + 5 = 0$

$(y-1)(y-5) = 0$

$$\Rightarrow y = \begin{cases} 1 \Rightarrow 5^x = 1 \Leftrightarrow x = 0 \\ 5 \Rightarrow 5^x = 5 \Leftrightarrow x = 1 \end{cases} \Rightarrow S = \{0;1\}$$

## Ex 4.2.1

b) $$3^{4x+2} - 36\cdot 3^{2x+1} = -243$$

$$3^{4x+2} - 36\cdot 3^{2x+1} + 243 = 0$$

$$3^{2(2x+1)} - \ldots \quad \ldots = 0$$

on pose $y = 3^{2x+1}$

$$\Rightarrow \quad y^2 - 36y + \underbrace{243}_{\substack{=3^5 \\ =9\cdot 27}} = 0 \qquad \Delta = 36^2 - 4\cdot 243 = \ldots$$

$$(y-9)(y-27) = 0$$

$$\Rightarrow \quad y = \begin{cases} 9 \Rightarrow 3^{2x+1} = 9 = 3^2 \Leftrightarrow 2x+1 = 2 \Leftrightarrow x = \frac{1}{2} \\ 27 \Rightarrow 3^{2x+1} = 27 = 3^3 \Leftrightarrow 2x+1 = 3 \Leftrightarrow x = 1 \end{cases}$$

$$\Rightarrow \quad S = \left\{ \frac{1}{2} ; 1 \right\}$$

l) $\underbrace{5\cdot 5^{4x-7}}_{5^{1+4x-7}} - 120\cdot 5^{2x-3} - 625 = 0$

$\Leftrightarrow \quad 5^{4x-6} - 120\cdot 5^{2x-3} - 625 = 0$

$y = 5^{2x-3}$

$\Rightarrow \quad y^2 - 120y - 625 = 0$

$\Leftrightarrow \quad (y-125)(y+5) = 0$

$\Leftrightarrow \quad y = \begin{cases} 125 \Leftrightarrow 5^{2x-3} = 5^3 \Leftrightarrow 2x-3=3 \Leftrightarrow x=3 \\ -5 \Leftrightarrow 5^{2x-3} = -5 \end{cases}$ ↯ impossible

$\Rightarrow \quad S = \{3\}$