

Prob 1 (2021)

a) $(x-1)^2 + (y-3)^2 + z^2 = 159 + 1 + 9 = 169 \Rightarrow \underline{C(1; 3; 0)}$ et $\underline{r=13}$ u

b) $P(5; 0; 12) \in d : \begin{cases} 5 = 5+k & \Leftrightarrow k=0 \\ 0 = k & \Leftrightarrow k=0 \checkmark \\ 12 = 12+k & \Leftrightarrow k=0 \end{cases}$

$P(5; 0; 12) \in \Gamma : 25 + 0 + 144 - 10 = 159 \checkmark$

$d \cap \Gamma : (5+k)^2 + k^2 + (12+k)^2 - 2(5+k) - 6 \cdot k = 159$

$$25 + 10k + k^2 + k^2 + 144 + 24k + k^2 - 10 - 2k - 6k = 159$$

$$3k^2 + 26k = 0$$

$$k(3k+26) = 0 \Leftrightarrow k=0 \text{ ou } k = -\frac{26}{3}$$

Q: $\begin{cases} x = 5 - 26/3 \\ y = -26/3 \\ z = 12 - 26/3 \end{cases} \Rightarrow \underline{Q\left(-\frac{11}{3}; -\frac{26}{3}; \frac{10}{3}\right)}$

c) $\Pi : (5-1)(x-1) + (0-3)(y-3) + 12z = 169$

$$4x - 4 - 3y + 9 + 12z = 169 \Leftrightarrow \underline{4x - 3y + 12z - 164 = 0}$$

d) α contient $d \Rightarrow \alpha$ passe par $D(5; 0; 12)$ et a comme v.d. $\vec{d} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

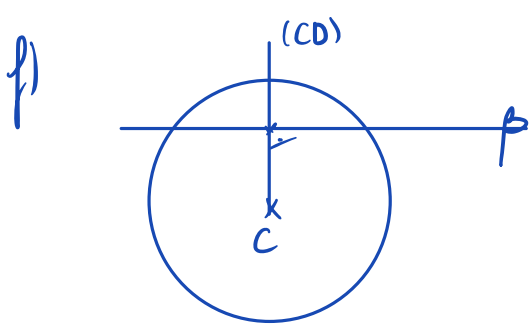
α contient $A \Rightarrow \vec{AD} = \begin{pmatrix} 1 \\ 12 \\ 11 \end{pmatrix}$ est un 2^e v.d.

$$\Rightarrow \underline{\alpha : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 12 \end{pmatrix} + l \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + m \begin{pmatrix} 1 \\ 12 \\ 11 \end{pmatrix}, \quad l, m \in \mathbb{R}}$$

e) $\alpha \perp \beta \Leftrightarrow \vec{n}_\alpha \perp \vec{n}_\beta \Leftrightarrow \vec{n}_\alpha \cdot \vec{n}_\beta = 0$

$$n_\alpha = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 12 \\ 11 \end{pmatrix} = \begin{pmatrix} -1 \\ -10 \\ 11 \end{pmatrix} \quad \text{et} \quad n_\beta = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \underline{n_\alpha \cdot n_\beta = -1 - 10 + 11 = 0 \checkmark}$$



$$(CD): \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$$

$$1) \{D\} = CD \cap \beta : 1 + \lambda + 3 + \lambda + \lambda - 17 = 0$$

$$3\lambda - 13 = 0 \quad \Leftrightarrow \quad \lambda = \frac{13}{3}$$

$$\Rightarrow : \begin{cases} x = 1 + 13/3 \\ y = 3 + 13/3 \\ z = 13/3 \end{cases} \Rightarrow \underline{D\left(\frac{16}{3}; \frac{22}{3}; \frac{13}{3}\right)}$$

$$2) \|\vec{CD}\| = \left\| \begin{pmatrix} 13/3 \\ 13/3 \\ 13/3 \end{pmatrix} \right\| = \sqrt{3 \cdot \frac{169}{9}} = \sqrt{\frac{169}{3}} = \frac{13 \cdot \sqrt{3}}{3}$$

$$\underline{r} = \sqrt{169 - \frac{169}{3}} = \underline{\sqrt{\frac{338}{3}}}$$