Exercice 1.6

Étudier l'ensemble de définition et le signe des fonctions suivantes :

a) $f(x) = \dfrac{x(x-5)}{(x+3)(x+5)^2}$

b) $f(x) = \dfrac{-x^2+x+6}{x^2-10x+25}$

c) $f(x) = \dfrac{6x^2-13x+6}{8x^3-36x^2+54x-27}$

d) $f(x) = \dfrac{x^3-5x^2+8x-4}{x^2-x-12}$

e) $f(x) = \dfrac{(x-2)^2(x+3)^3 - 2(x-2)^3(x+3)^2}{(x-4)^2}$

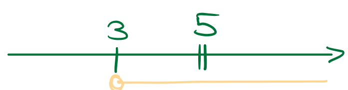
f) $f(x) = x + 2 + \dfrac{2}{x-1}$

g) $f(x) = 4 - \dfrac{3}{x} + \dfrac{3}{x-3}$

h) $f(x) = \dfrac{\sqrt{x-3}}{x-5}$

cond : $x-3 \geqslant 0$ et $x-5 \neq 0$

$x \geqslant 3$ et $x \neq 5$



$ED(f) = [3\,;+\infty[- \{5\}$

zéro : $\sqrt{x-3} = 0 \Leftrightarrow x-3 = 0 \Leftrightarrow x = 3$

x		3		5	
sgn(f)	////	0	$-$	$\|\|$	$+$

i) $f(x) = \dfrac{x^2+x-12}{\sqrt{x^2-4}} = \dfrac{(x+4)(x-3)}{\sqrt{(x+2)(x-2)}}$

cond : $\underbrace{x^2-4 \geq 0 \text{ et } \sqrt{x^2-4} \neq 0}$

$x^2-4>0$

$(x+2)(x-2)>0$

x		-2		2	
x^2-4	$+$	0	$-$	0	$+$

$ED(f) =]-\infty;-2[\cup]2;+\infty[$

zéro : -4 et 3 et v.i : $[-2;2]$

signe :

x		-4		-2		2		3	
sgn(f)	$+$	0	$-$	$\|$	//////	$\|$	$-$	0	$+$

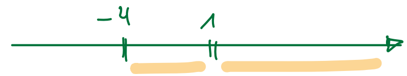
$f(1000): \dfrac{+}{+}$

j) $f(x) = \dfrac{\sqrt{3-x}}{(x-1)\cdot\sqrt{x+4}}$

cond : $x-1 \neq 0$ et $\underbrace{x+4 \geq 0 \text{ et } \sqrt{x+4} \neq 0}$

$x \neq 1$ et $x+4>0$

$x>-4$



$ED(f) =]-4;+\infty[- \{1\}$

zéro : $\sqrt{3-x} = 0 \Leftrightarrow 3-x=0 \Leftrightarrow x=3$

signe :

x		-4		1		3	
f	///	$\|$	$-$	$\|$	$+$	0	$+$

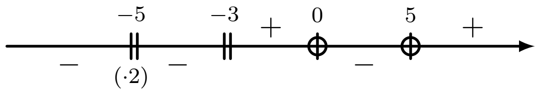
ne dépend que du signe de $x-1$ car $\sqrt{3-x}$ et $\sqrt{x+4}$ sont tjrs pos. sur $ED(f)$

k) $f(x) = \dfrac{1-\sqrt{x}}{2-\sqrt{x}}$

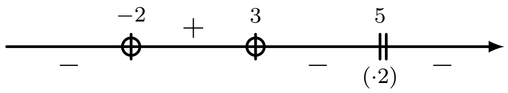
l) $f(x) = \dfrac{\sqrt{x^2-1}-\sqrt{x+1}}{3-x}$

1.6

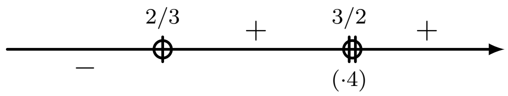
a) $ED(f) = \mathbb{R} - \{-5\,;-3\}$



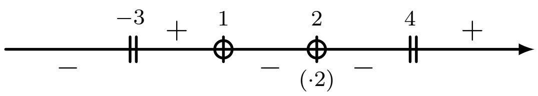
b) $ED(f) = \mathbb{R} - \{5\}$



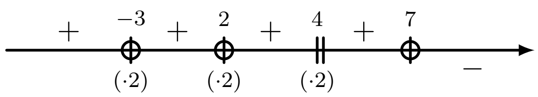
c) $ED(f) = \mathbb{R} - \left\{\frac{3}{2}\right\}$



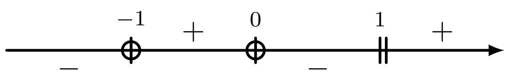
d) $ED(f) = \mathbb{R} - \{-3\,;4\}$



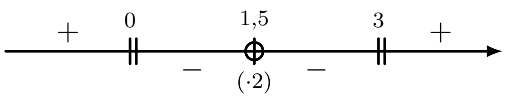
e) $ED(f) = \mathbb{R} - \{4\}$



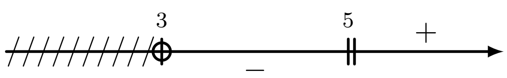
f) $ED(f) = \mathbb{R} - \{1\}$



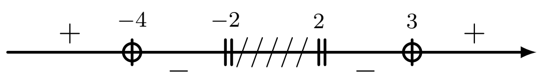
g) $ED(f) = \mathbb{R} - \{0\,;3\}$



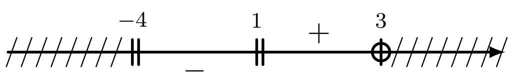
h) $ED(f) = [3\,;+\infty[- \{5\}$



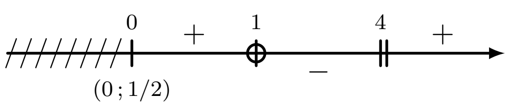
i) $ED(f) =]-\infty\,;-2[\cup]2\,;+\infty[$



j) $ED(f) =]-4\,;3] - \{1\}$



k) $ED(f) = \mathbb{R}_+ - \{4\}$



l) $ED(f) = \{-1\} \cup [1\,;3[\cup]3\,;+\infty[$