Exemple d'étude de fonction

$$f(x) = \frac{(2x+1)(x-1)(x-2)}{(x+1)(x-1)}$$

zéros : $-\frac{1}{2}$; (1) ; 2

v.i. : ± 1

$\notin ED(f)$

$$\begin{cases} (2x+1)(x-1)(x-2) \\ = (2x^2-2x+x-1)(x-2) \\ = (2x^2-x-1)(x-2) \\ = 2x^3-4x^2-x^2+2x-x+2 \\ = 2x^3-5x^2+x+2 \end{cases}$$

1) $ED(f) = \mathbb{R} - \{\pm 1\}$

2) signe :

x	-1		$-\frac{1}{2}$		1		2	
$\text{sgn}(f)$	$-$	$\|\|$	$+$	0	$-$	$\|\|$	$-$	0 $+$
					(2)			

$f(1000) : \frac{+}{+}$

3) asymptotes

AV/trou : $\lim_{x \to -1} f(x) = \frac{\text{“}-1 \cdot (-2) \cdot (-3)\text{”}}{0} = \infty \Rightarrow x = -1$ AV

$$\lim_{x \to 1} f(x) = \text{“}\frac{0}{0}\text{”} = \lim_{x \to 1} \frac{(2x+1)(x-2)}{x+1} = \frac{3 \cdot (-1)}{2} = -\frac{3}{2}$$

$\Rightarrow$ “trou” $(1 ; -\frac{3}{2})$

AH/AO : $f(x) = \frac{2x^3 \ldots}{x^2-1}$ $\quad Deg(N) = Deg(D)+1$ $\quad 3 = 2+1$

AO : $f(x) = \frac{2x^3-5x^2+x+2}{x^2-1}$

$$\begin{array}{r|l} 2x^3-5x^2+x+2 & x^2-1 \\ -2x^3 \quad +2x & \hline \\ -5x^2+3x+2 & 2x-5 \\ +5x^2 \quad -5 & \\ \hline 3x-3 & \end{array}$$

$$f(x) = 2x-5+\frac{3x-3}{x^2-1}$$

$\Rightarrow$ $y = 2x-5$ est AO

graphe :