Ex 1.2.28

$$U = \langle \underbrace{4x^2 - x + 2}_{p_1} \,;\, \underbrace{2x^2+6x+3}_{p_2} \,;\, \underbrace{-4x^2+10x+2}_{p_3} \rangle \subset P_3 = \mathbb{R}_3[x]$$

$$(\dim = 4)$$

$\dim(U) \leqslant 4$

p_1, p_2, p_3 libres ?

$$\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 = 0$$

$$\Leftrightarrow \lambda_1(4x^2 - x + 2) + \lambda_2(2x^2+6x+3) + \lambda_3(-4x^2+10x+2) = 0x^3 + 0x^2 + 0x + 0$$

$$\Leftrightarrow 0x^3 + (4\lambda_1 + 2\lambda_2 - 4\lambda_3)x^2 + (-\lambda_1 + 6\lambda_2 + 10\lambda_3)x + (2\lambda_1 + 3\lambda_2 + 2\lambda_3) = 0$$

$$\Rightarrow \begin{cases} 4\lambda_1 + 2\lambda_2 - 4\lambda_3 = 0 \\ -\lambda_1 + 6\lambda_2 + 10\lambda_3 = 0 \\ 2\lambda_1 + 3\lambda_2 + 2\lambda_3 = 0 \end{cases}$$

comme $\begin{vmatrix} 4 & 2 & -4 \\ -1 & 6 & 10 \\ 2 & 3 & 2 \end{vmatrix} = 4(12-30) + (4+12) + 2(20+24) = 32 \neq 0$

La solution du système est unique $\Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0$

$\Rightarrow$ les 3 vecteurs sont libres et engendrent U, ils forment donc une base $\Rightarrow$ $\dim(U) = 3$

Variante :

$$p_1 = 0\cdot x^3 + 4\cdot x^2 - 1\cdot x + 2 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 2 \end{pmatrix}_B \quad p_2 = \begin{pmatrix} 0 \\ 2 \\ 6 \\ 3 \end{pmatrix}_B \quad \text{et} \quad p_3 = \begin{pmatrix} 0 \\ -4 \\ 10 \\ 2 \end{pmatrix}_B$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 4 & 2 & -4 \\ -1 & 6 & 10 \\ 2 & 3 & 2 \end{pmatrix} \underset{-L_3 \leftrightarrow L_2}{\sim} \begin{pmatrix} 0 & 0 & 0 \\ 1 & -6 & -10 \\ 4 & 2 & -4 \\ 2 & 3 & 2 \end{pmatrix} \underset{\substack{L_3 - 4L_2 \to L_3 \\ L_4 - 2L_2 \to L_4}}{\sim} \begin{pmatrix} 0 & 0 & 0 \\ 1 & -6 & -10 \\ 0 & 13 & 18 \\ 0 & 15 & 22 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0 & 0 & 0 \\ 1 & -6 & -10 \\ 0 & 1 & 18/13 \\ 0 & 15 & 22 \end{pmatrix} \underset{\substack{L_2 + 6L_3 \to L_2 \\ L_4 - 15L_3 \to L_4}}{\sim} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -22/13 \\ 0 & 1 & 18/13 \\ 0 & 0 & 16/13 \end{pmatrix} \underset{\frac{13}{16}L_4 \to L_4}{\sim}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -22/13 \\ 0 & 1 & 18/13 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} L_2 + \frac{22}{13} L_4 \to L_2 \\ L_3 - \frac{18}{13} L_4 \to L_4 \end{array} \sim \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{libres}}$$