2.7.5 Déterminer, suivant les valeurs de l'entier naturel n, les asymptotes de la fonction f définie par $f(x) = \dfrac{x^n + 3}{x^2 - 9}$.

$\underline{n=0}$: $f(x) = \dfrac{x^0+3}{x^2-9} = \dfrac{4}{(x+3)(x-3)}$ $\qquad \underline{\underline{ED = \mathbb{R} - \{\pm 3\}}}$

$\underline{\text{AV/hou}}$: $\lim\limits_{x \to 3} f(x) = "\dfrac{4}{0}" = \infty \Rightarrow x = 3$ AV

$\lim\limits_{x \to -3} f(x) = "\dfrac{4}{0}" = \infty \Rightarrow x = -3$ AV

$\underline{\text{AH/AO}}$ $\lim\limits_{x \to \infty} f(x) = \dfrac{4}{\infty} = 0 \Rightarrow y = 0$ AH

$\underline{n=1}$ $f(x) = \dfrac{x+3}{(x+3)(x-3)}$

$\underline{\text{AV/hou}}$ $\lim\limits_{x \to 3} f(x) = "\dfrac{6}{0}" = \infty \Rightarrow x = 3$ AV

$\lim\limits_{x \to -3} f(x) = "\dfrac{0}{0}" = \lim\limits_{x \to -3} \dfrac{1}{x-3} = \dfrac{1}{-6} \Rightarrow$ "hou"

$\left(-3 ; -\frac{1}{6}\right)$

$\underline{\text{AH/AO}}$: $\lim\limits_{x \to \infty} f(x) \overset{"\frac{\infty}{\infty}"}{=} \lim\limits_{x \to \infty} \dfrac{\cancel{x}^{\,1}}{x^{\cancel{2}}} = \dfrac{1}{\infty} = 0 \Rightarrow y = 0$ AH

$\underline{n=2}$ $f(x) = \dfrac{x^2+3}{x^2-9}$

$\underline{\text{AV/hou}}$: $\lim_{x \to -3} f(x) = \text{"}\dfrac{12}{0}\text{"} = \infty \Rightarrow x = -3$ AV

$\lim_{x \to +3} f(x) = \text{"}\dfrac{12}{0}\text{"} = \infty \Rightarrow x = 3$ AV

$\underline{\text{AH/AO}}$ $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \dfrac{\cancel{x^2}^{\,1}}{\cancel{x^2}_{\,1}} = 1 \Rightarrow$ AH : $y = 1$

$\underline{n=3}$ $f(x) = \dfrac{x^3+3}{x^2-9}$

$\underline{\text{AV/hou}}$: . . . AV : $x = -3$ et $x = 3$

$\underline{\text{AH/AO}}$: $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \dfrac{x^{\cancel{3}}}{\cancel{x^2}_{\,1}} = \infty$ pas d'AH

$\text{Deg}(N) = \text{Deg}(D) + 1$

$$\begin{array}{r|l} x^3+3 & x^2-9 \\ \cline{2-2} -(x^3 - 9x) & x \\ \cline{1-1} 9x+3 & \end{array}$$

$\Rightarrow f(x) = x + \dfrac{9x+3}{x^2-9}$

$\Rightarrow$ AO : $y = x$

<u>si $n>3$</u> $$f(x) = \frac{x^n+3}{x^2-9}$$

AV : $x=-3$ et $x=3$

<u>AH/AO</u> $$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^n}{x^2} = \lim_{x\to\infty} x^{n-2} = \infty$$

$\Rightarrow$ pas d'AH

AO : non $\quad Deg(N) \neq Deg(D)+1$

<u>En résumé</u> :

AH si $Deg(N) \leq Deg(D)$

AO si $Deg(N) = Deg(D)+1$

ni AH ni AO ; si $Deg(N) > Deg(D)+1$