Dérivée (suite)

Exemple 4) $f(x) = x^3$

$$f'(a) = \lim_{x \to a} \frac{x^3 - a^3}{x - a} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to a} \frac{\cancel{(x-a)}(x^2 + ax + a^2)}{\cancel{x-a}}$$

$$= \lim_{x \to a} (x^2 + ax + a^2) = a^2 + a \cdot a + a^2 = a^2 + a^2 + a^2 = 3a^2$$

$$\Rightarrow f'(x) = 3x^2$$

5) $f(x) = x^4$

$$f'(a) = \lim_{x \to a} \frac{x^4 - a^4}{x - a} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to a} \frac{(x^2 + a^2)\overbrace{(x^2 - a^2)}^{(x+a)\cancel{(x-a)}}}{\cancel{x-a}}$$

$$= \lim_{x \to a} (x^2 + a^2)(x + a) = (a^2 + a^2)(a + a) = 2a^2 \cdot 2a = 4a^3$$

$$\Rightarrow f'(x) = 4x^3$$

Dérivée de fonctions usuelles

1. fct constante

2.

$$\boxed{\begin{aligned}(k)' &= 0\\ (x^n)' &= n \cdot x^{n-1}\end{aligned}}$$

exples : $(x^2)' = 2x$ $\quad$ $(x^6)' = 6x^5$

$(x^3)' = 3x^2$ $\quad$ $(x^{100})' = 100x^{99}$

$\to$ $\left(\frac{1}{x^2}\right)' = \left(x^{-2}\right)' = -2x^{-3} = -2 \cdot \frac{1}{x^3} = -\frac{2}{x^3}$

$\to$ $\left(\sqrt[3]{x}\right)' = \left(x^{1/3}\right)' = \frac{1}{3} x^{\frac{1}{3}-1} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{x^{2/3}}$

$= \frac{1}{3\sqrt[3]{x^2}}$

$\boxed{\left(\frac{1}{x}\right)'} = \left(x^{-1}\right)' = -1 \cdot x^{-2} = -1 \cdot \frac{1}{x^2} = \boxed{-\frac{1}{x^2}}$

$\boxed{\left(\sqrt{x}\right)'} = \left(x^{1/2}\right)' = \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} = \boxed{\frac{1}{2\sqrt{x}}}$

Règles de dérivation

Soit $u(x)$ et $v(x)$ deux fonctions et $k \in \mathbb{R}$

1. $f(x) = k \cdot u(x) \Rightarrow f'(x) = k \cdot u'(x)$

   exemple : $(x^5)' = 5x^4$

   $(3x^5)' = 3 \cdot 5x^4 = 15x^4$

2. $f(x) = u(x) + v(x) \Rightarrow f'(x) = u'(x) + v'(x)$

   exemple : $(x^5 + x^3)' = 5x^4 + 3x^2$

formulaire

1. $\boxed{(ku)' = k \cdot u'}$

2. $\boxed{(u+v)' = u' + v'}$

Exemple :

$$f(x) = 3x^5 + 2x^4 + x^3 - 5x^2 + x + 2$$

$$f'(x) = 15x^4 + 2 \cdot 4x^3 + 3x^2 - 5 \cdot 2x + 1 \cdot x^0 + 0$$

$$= 15x^4 + 8x^3 + 3x^2 - 10x + 1$$