Règle de dérivation (suite)

soit u et v des fonctions de x.

3) $\boxed{(u \cdot v)' = u' \cdot v + u \cdot v'}$

4) $\boxed{\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}}$

form. p. 17

Exemples

a) $f(x) = \underbrace{(x+5)}_{u} \cdot \underbrace{(2x-1)}_{v}$

$u = x+5$ $\quad$ $v = 2x-1$

$u' = 1$ $\quad$ $v' = 2$

$$f'(x) = 1 \cdot (2x-1) + (x+5) \cdot 2$$
$$= 2x - 1 + 2x + 10$$
$$= 4x + 9$$

b) $f(x) = (3x^2-5)(4x-1)$

$$\begin{cases} u = 3x^2-5 & v = 4x-1 \\ u' = 3\cdot 2x = 6x & v' = 4 \end{cases}$$

$$f'(x) = 6x(4x-1) + (3x^2-5)\cdot 4$$

$$= 24x^2 - 6x + 12x^2 - 20 \quad = \quad 36x^2 - 6x - 20$$

c) $f(x) = (x^2-1)\sqrt{x}$

$$\begin{cases} u = x^2-1 & v = \sqrt{x} \\ u' = 2x & v' = \dfrac{1}{2\sqrt{x}} \end{cases}$$

$$f'(x) = 2x\cdot\sqrt{x} + (x^2-1)\cdot\frac{1}{2\sqrt{x}}$$

$$= 2x\sqrt{x} + \frac{x^2-1}{2\sqrt{x}} \quad !$$

$$= \frac{2x\sqrt{x}\cdot 2\sqrt{x}}{2\sqrt{x}} + \frac{x^2-1}{2\sqrt{x}} = \frac{4x\cdot x + x^2-1}{2\sqrt{x}} = \frac{5x^2-1}{2\sqrt{x}}$$

d) $f(x) = \underset{u}{(x^2-4)} \cdot \underset{v}{(1-x)} \cdot \underset{w}{(1+x)}$

$$u = x^2-4 \qquad v = 1-x \qquad w = 1+x$$
$$u' = 2x \qquad v' = -1 \qquad w' = 1$$

$$f'(x) = \underset{u'\cdot v\cdot w}{2x(1-x)(1+x)} + \underset{u\cdot v'\cdot w}{(x^2-4)\cdot(-1)\cdot(1+x)} + \underset{u\cdot v\cdot w'}{(x^2-4)(1-x)\cdot 1}$$

$$= 2x(1-x^2) - (x^2+x^3-4-4x) + (x^2-x^3-4+4x)$$

$$= 2x - 2x^3 - \cancel{x^2} - x^3 + \cancel{4} + 4x + \cancel{x^2} - x^3 - \cancel{4} + 4x$$

$$= -4x^3 + 10x$$

e) $f(x) = \frac{x+5}{2x-1}$

$u = x+5$ $\quad v = 2x-1$

$u' = 1$ $\quad v' = 2$

$$f'(x) = \frac{1\cdot(2x-1) - (x+5)\cdot 2}{(2x-1)^2} = \frac{2x-1-2(x+5)}{(2x-1)^2} = \frac{-6}{(2x-1)^2}$$

f) $f(x) = \frac{2x}{x^2-1}$

$u = 2x$ $\quad v = x^2-1$

$u' = 2$ $\quad v' = 2x$

$$f'(x) = \frac{2(x^2-1) - \overbrace{2x\cdot 2x}^{4x^2}}{(x^2-1)^2} = \frac{-2x^2-2}{(x^2-1)^2} = \frac{-2(x^2+1)}{(x^2-1)^2}$$

g) $f(x) = \frac{1}{x^2+3}$

$u = 1$ $\quad v = x^2+3$

$u' = 0$ $\quad v' = 2x$

$$f'(x) = \frac{0\cdot(x^2+3) - 1\cdot 2x}{(x^2+3)^2} = \frac{-2x}{(x^2+3)^2}$$

$$\Rightarrow \boxed{\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}}$$

+ ex 2.8.8