

Ex 1.3.10

a) $h: \mathbb{R}^2 \rightarrow \mathbb{R}$

$(x, y) \mapsto x+y$

$(1, 0) \mapsto 1$

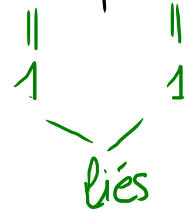
$(0, 1) \mapsto 1$

$H = \begin{pmatrix} 1 & 1 \end{pmatrix}$

* $\text{Ker}(h) : \underline{h((x, y))} = 0 \Leftrightarrow x+y = 0$
 $\Leftrightarrow y = -x$

$\Rightarrow \text{Ker}(h) = \{(x, -x) \mid x \in \mathbb{R}\}$
 $= \{x(1, -1) \mid x \in \mathbb{R}\}$
 $= \langle (1, -1) \rangle \quad \leftarrow \dim(\text{Ker}(h)) = 1$

* $\text{Im}(h) = \langle h(e_1), h(e_2) \rangle = \langle h(e_1) \rangle = \langle 1 \rangle = \mathbb{R}$



ou

thm du rang : $\dim(\mathbb{R}^2) = \dim(\text{Ker}(h)) + \underbrace{\dim(\text{Im}(h))}_{\text{rang}(h)}$

$2 = 1 + \text{rang}(h)$

$\Rightarrow \text{rang}(h) = 1$

comme $\text{Im}(h)$ est un SEV de $\mathbb{R} \Rightarrow \text{Im}(h) = \mathbb{R}$
 $\uparrow \dim=1 \quad \quad \quad \uparrow \dim=1$

$$c) \quad h: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (2x - y, x)$$

$$H = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\text{car } \underline{h(1, 0) = (2, 1)}$$

$$\underline{h(0, 1) = (-1, 0)}$$

* $\text{Ker}(h)$: On cherche (x, y) tel que

$$(2x - y, x) = (0, 0)$$

$$\begin{cases} 2x - y = 0 \\ x = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$\Rightarrow \text{Ker}(h) = \{(0, 0)\}$$

$$= \langle (0, 0) \rangle$$

$$(\dim(\text{Ker}(h)) = 0)$$

* $\text{Im}(h)$: $\text{rang}(h) = \dim(\text{Im}(h)) = \underset{\substack{\uparrow \\ \text{dim} \\ \text{ens. de départ.}}}{2} - 0 = 2 = \dim(\mathbb{R}^2) = \underset{\substack{\uparrow \\ \text{ens. d'arrivée}}}{2}$

$$\Rightarrow \text{Im}(h) = \mathbb{R}^2$$

ou $\text{Im}(h) = \langle h(e_1), h(e_2) \rangle = \langle (2, 1), (-1, 0) \rangle$

$\swarrow \quad \searrow$
libres

$$= \langle (1, 0), (0, 1) \rangle = \mathbb{R}^2$$

\uparrow
car même dimension.