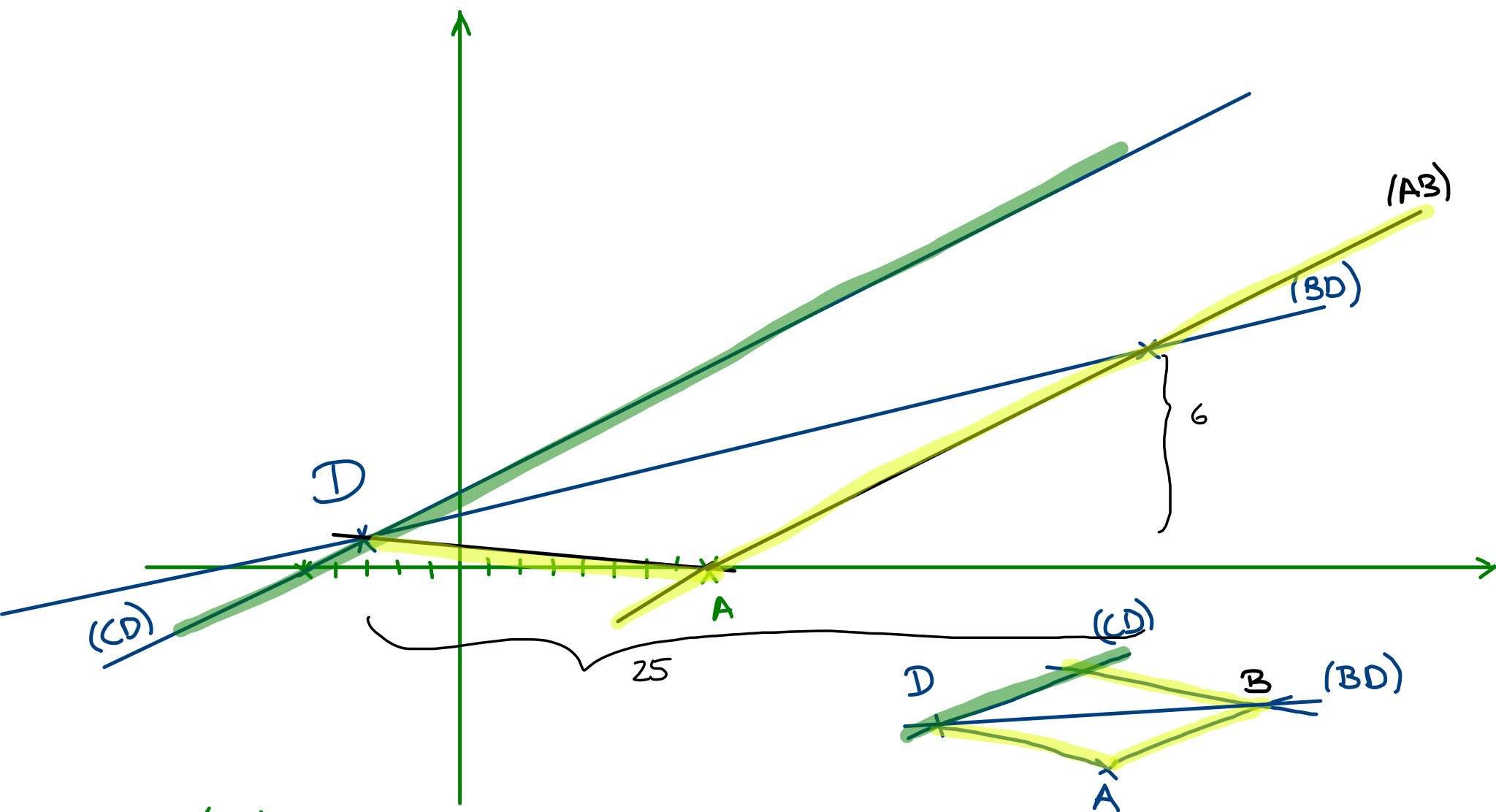


3.1.15 D'un parallélogramme $ABCD$, on donne le sommet $A(8;0)$, l'équation du côté $CD : x - 2y + 5 = 0$, ainsi que l'équation de la diagonale $BD : 6x - 25y = -43$. Déterminer les équations cartésiennes des côtés AB , AD et BC .



$$(CD): x - 2y + 5 = 0 \quad m = \frac{1}{2}$$

$$\text{si } y = 0 \Rightarrow x + 5 = 0 \quad x = -5 \Rightarrow (-5; 0)$$

$$(BD): 6x - 25y + 43 = 0 \quad m = \frac{6}{25}$$

$$D = (CD) \cap (BD) \quad \begin{cases} x - 2y = -5 & \cdot 25 \\ 6x - 25y = -43 & \cdot (-2) \end{cases}$$

$$\Rightarrow + \begin{array}{r} 25x - 50y = -125 \\ -12x + 50y = 86 \end{array}$$

$$\begin{array}{r} 13x = -39 \\ x = -3 \end{array} \quad (1^e)$$

$$\Rightarrow -3 - 2y = -5$$

$$-2y = -2$$

$$y = 1$$

$$\Rightarrow D(-3; 1)$$

$$A(8;0) \quad D(-3;1)$$

$$1) \quad (AD): \text{ vecteur directeur } \overrightarrow{AD} = \begin{pmatrix} -3-8 \\ 1-0 \end{pmatrix} = \begin{pmatrix} -11 \\ 1 \end{pmatrix}$$

$$\Rightarrow a = 1 \quad \text{et} \quad b = 11 \quad \Rightarrow$$

$$(AD): \quad x + 11y + c = 0$$

$$A(8;0) \in (AD) \Rightarrow 8 + c = 0$$

$$c = -8$$

$$\Rightarrow \underline{(AD): x + 11y - 8 = 0}$$

$$2) \quad (AB) // (DC) \Rightarrow \left. \begin{array}{l} (AB): x - 2y + c = 0 \\ \downarrow \\ x - 2y + 5 = 0 \end{array} \right\} \Rightarrow$$

$$A(8;0) \in (AB): \quad 8 + c = 0$$

$$c = -8$$

$$\underline{(AB): x - 2y - 8 = 0}$$

On cherche B

$$B = (BD) \cap (AB) : \begin{cases} 6x - 25y = -43 & | \cdot 2 & | \cdot 1 \\ x - 2y = 8 & | \cdot (-25) & | \cdot (-6) \end{cases}$$

$$\begin{array}{r} + \quad 12x - 50y = -86 \\ \quad -25x + 50y = -200 \\ \hline -13x \quad = -286 \\ \quad x \quad = 22 \end{array}$$

$$\begin{array}{r} 6x - 25y = -43 \\ -6x + 12y = -48 \\ \hline -13y = -91 \\ \quad y = 7 \end{array}$$

$$\Rightarrow B(22; 7)$$

$$\left. \begin{array}{l} 3) (BC) \parallel (AD) \Rightarrow (BC): x + my + c = 0 \\ B \in (BC) \Rightarrow \begin{array}{l} 22 + 77 + c = 0 \\ c = -99 \end{array} \end{array} \right\} \Rightarrow \underline{(BC): x + my - 99 = 0}$$