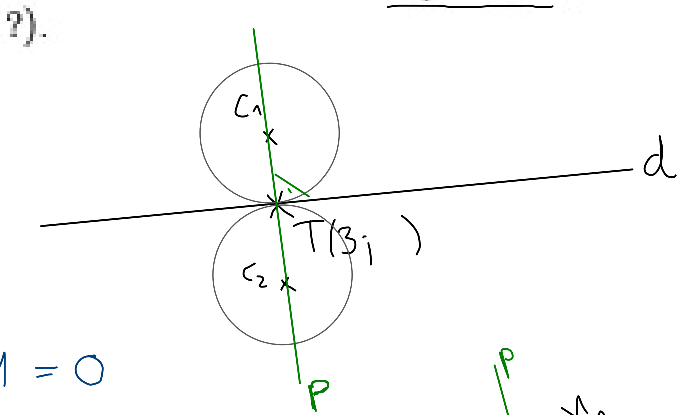
2.1.11 Déterminer les équations des cercles de rayon $\sqrt{5}$ qui sont tangents à la droite $x - 2y = 1$ au point $T(3; ?)$.

$d: \ x - 2y - 1 = 0$

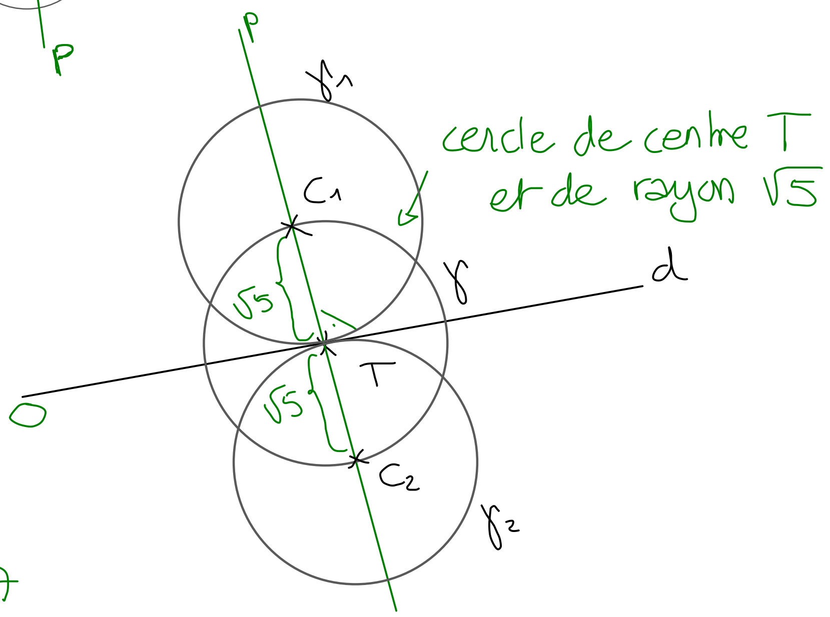


1) $T \in d \Rightarrow 3 - 2y - 1 = 0$

$-2y = -2$

$y = 1$

$\Rightarrow T(3;1)$



2) $p \perp d \Rightarrow p: \ 2x + y + c = 0$

$T \in p: \quad 2 \cdot 3 + 1 + c = 0$

$c = -7$

$\Rightarrow p: \ 2x + y - 7 = 0$

3) cercle de centre T et de rayon $\sqrt{5}$: $\gamma: (x-3)^2 + (y-1)^2 = 5$

4) C_1 et C_2 : $p \cap \gamma$: $\begin{cases} 2x + y - 7 = 0 \\ (x-3)^2 + (y-1)^2 = 5 \end{cases} \Leftrightarrow y = -2x + 7$

$\Rightarrow (x-3)^2 + (\underbrace{-2x + 7 - 1}_{(-2x+6)^2})^2 = 5$

$\Rightarrow x^2 - 6x + 9 + 4x^2 - 24x + 36 = 5$

$\Leftrightarrow 5x^2 - 30x + 40 = 0$

$\Leftrightarrow x^2 - 6x + 8 = 0$

$\Leftrightarrow (x-4)(x-2) = 0$

$\Leftrightarrow$ $x = 4 \Rightarrow y = -2 \cdot 4 + 7 = -1 \Rightarrow C_1(4; -1)$

 $x = 2 \Rightarrow y = -2 \cdot 2 + 7 = 3 \Rightarrow C_2(2; 3)$

5) $$\boxed{\begin{aligned} \gamma_1 &: (x-4)^2 + (y+1)^2 = 5 \\ \gamma_2 &: (x-2)^2 + (y-3)^2 = 5 \end{aligned}}$$