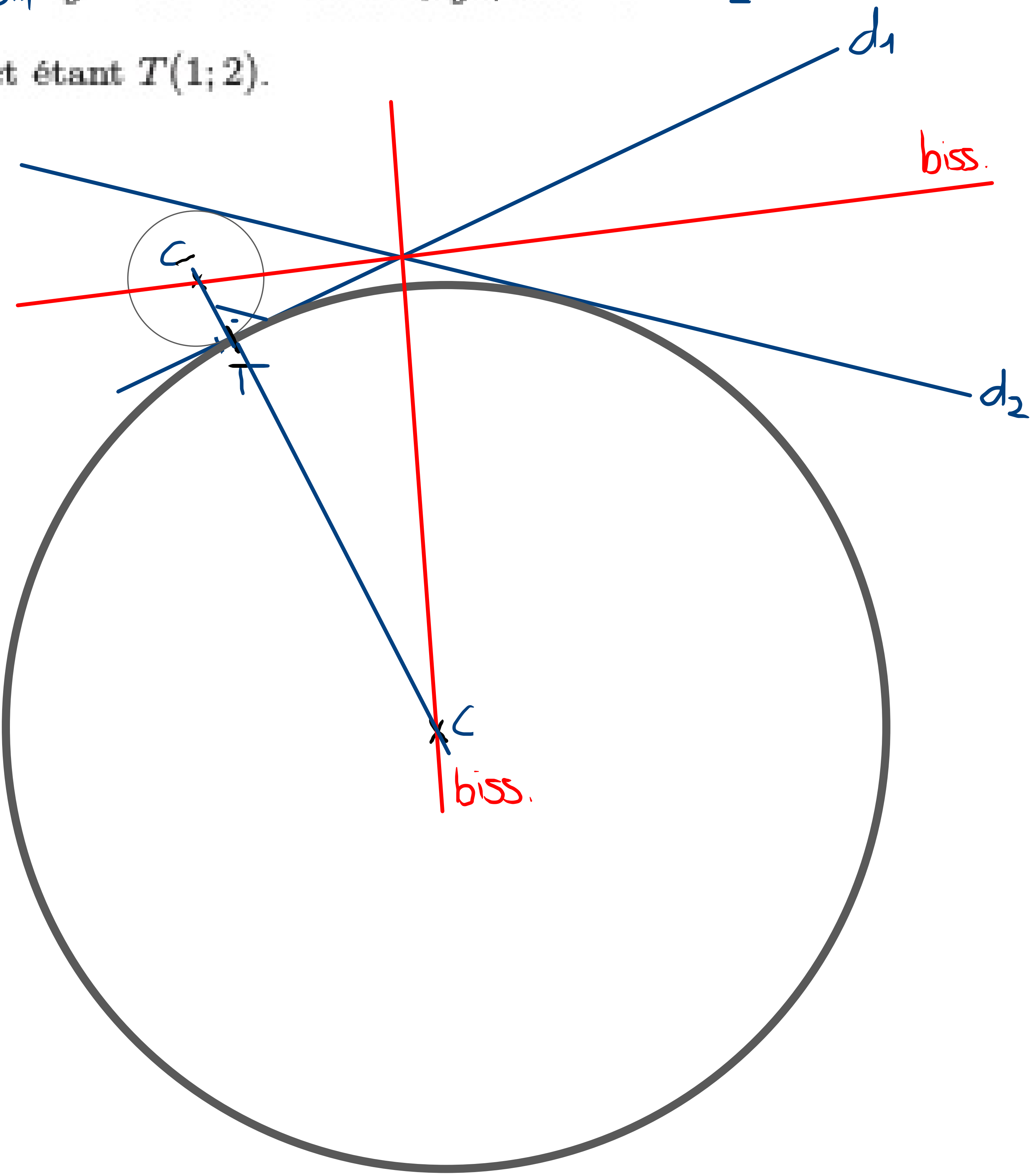
5. 12. Déterminer les équations des cercles tangents aux droites

$$d_1 : y = 7x - 5 \quad \text{et} \quad x + y + 13 = 0 \ : d_2$$

l'un des points de contact étant $T(1;2)$.

$T \in d_1 : \ 2 = 7 - 5$ ✓



bissectrices : $d_1: 7x - y - 5 = 0$ $\quad d_2: x + y + 13 = 0$

$$\frac{7x - y - 5}{\underbrace{\sqrt{7^2 + (-1)^2}}_{\underbrace{\sqrt{50}}_{= 5\sqrt{2}}}} = \pm \frac{x + y + 13}{\underbrace{\sqrt{1^2 + 1^2}}_{\sqrt{2}}} \quad \Big| \cdot 5\sqrt{2}$$

$$\Leftrightarrow \quad 7x - y - 5 = \pm 5(x + y + 13)$$

$$\Leftrightarrow \quad 7x - y - 5 = \begin{cases} 5x + 5y + 65 \\ -5x - 5y - 65 \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} 2x - 6y - 70 = 0 \Leftrightarrow x - 3y - 35 = 0 : b_1 \\ 12x + 4y + 60 = 0 \Leftrightarrow 3x + y + 15 = 0 : b_2 \end{cases}$$

On cherche (CT) qui est perpendiculaire à $d_1 : 7x - y - 5 = 0$

$$\left.\begin{array}{l} \Rightarrow \ (CT) : x + 7y + c = 0 \\ \text{et } T(1;2) \in (CT) \Rightarrow 1 + 7 \cdot 2 + c = 0 \\ \qquad\qquad\qquad\qquad\qquad c = -15 \end{array}\right\} \Rightarrow (CT) : x + 7y - 15 = 0$$

$$C_1 = (CT) \cap b_1 : \begin{cases} x + 7y = 15 \\ x - 3y = 35 \end{cases} \begin{vmatrix} 1 \\ -1 \end{vmatrix} \Rightarrow 10y = -20$$
$$y = -2$$

$$\Rightarrow x + 7(-2) = 15$$
$$x = 29 \Rightarrow C_1(29;-2)$$

et $r = \delta(C;T) = \|\vec{CT}\|$

ou $r = \delta(C;d_1)$ ou $\delta(C;d_2)$

ou

$$\gamma_1 : (x-29)^2 + (y+2)^2 = r^2$$
$$T \in \gamma_1 : (1-29)^2 + (2+2)^2 = 800 \qquad (\Rightarrow r = \sqrt{800})$$

$$\Rightarrow \underline{\gamma_1 : (x-29)^2 + (y+2)^2 = 800}$$

2^e cercle

$$\{C_2\} = (CT) \cap b_2 : \begin{cases} x + 7y = 15 \\ 3x + y = -15 \end{cases} \Leftrightarrow \dots \Rightarrow C_2(-6;3)$$

$$\Rightarrow \gamma_2 : (x+6)^2 + (y-3)^2 = r^2$$

$$T \in \gamma_2 : (1+6)^2 + (2-3)^2 = 49 + 1 = 50 \qquad (r = \sqrt{50} = 5\sqrt{2})$$

$$\Rightarrow \gamma_2 : (x+6)^2 + (y-3)^2 = 50$$