Ex 1.1.5

$(k)' = 0$ avec k un nombre comme e^2 par exple

a) $\lim\limits_{x \to 2} \dfrac{e^x - e^2}{x-2} = \dfrac{e^2 - e^2}{2-2} = \text{"}\dfrac{0}{0}\text{"} \overset{BH}{=} \lim\limits_{x \to 2} \dfrac{e^x}{1} = \dfrac{e^2}{1} = \underline{e^2}$

c) $\lim\limits_{x \to 0} \dfrac{x e^x}{1 - e^x} = \text{"}\dfrac{0}{0}\text{"} \overset{B.H}{=} \lim\limits_{x \to 0} \dfrac{1 \cdot e^x + x \cdot e^x}{-e^x} = \dfrac{1+0}{-1} = \underline{-1}$

f) $\lim\limits_{x \to +\infty} \dfrac{2e^x - 1}{e^x + 2} = \text{"}\dfrac{+\infty}{+\infty}\text{"} \overset{B.H.}{=} \lim\limits_{x \to +\infty} \dfrac{2\cancel{e^x}}{\cancel{e^x}_1} = \underline{2}$

h) $\lim\limits_{x \to +\infty} \dfrac{e^x}{x^2 - 2x + 3} = \underset{f.i.}{\text{"}\dfrac{+\infty}{+\infty}\text{"}} \overset{B.H.}{=} \lim\limits_{x \to +\infty} \dfrac{e^x}{2x-2} = \underset{f.i.}{\text{"}\dfrac{+\infty}{+\infty}\text{"}} \overset{BH}{=} \lim\limits_{x \to +\infty} \dfrac{e^x}{2}$

$= \dfrac{+\infty}{2} = \underline{+\infty}$

Ex 1.1.9

a) $$\lim_{x\to 0} \frac{\ln(x+1)}{x} = \underset{\text{f.i.}}{\text{"}\frac{0}{0}\text{"}} \overset{BH}{=} \lim_{x\to 0} \frac{\frac{1}{x+1}}{1} = 1$$

c) $$\lim_{x\to e} \frac{\ln(x)-1}{x-e} = \underset{\text{f.i.}}{\text{"}\frac{0}{0}\text{"}} \overset{BH}{=} \lim_{x\to e} \frac{\frac{1}{x}}{1} = \frac{1}{e}$$

d) $$\lim_{x\to 2} \frac{\ln(x^2-3)}{2-x} = \text{"}\frac{0}{0}\text{"} \overset{BH}{=} \lim_{x\to 2} \frac{\frac{2x}{x^2-3}}{-1} = \lim_{x\to 2} -\frac{2x}{x^2-3}$$

$$= -\frac{4}{1} = -4$$

e) $$\lim_{x\to -\infty} \frac{\ln(x^2+1)}{x} \underset{BH}{\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim_{x\to -\infty} \frac{\frac{2x}{x^2+1}}{1} = \lim_{x\to -\infty} \frac{2x}{x^2+1} \underset{BH}{\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}}$$

$$= \lim_{x\to -\infty} \frac{\cancel{2}}{\cancel{2}x} = \frac{1}{-\infty} = 0$$

f) $$\lim_{x\to +\infty} \frac{\ln(x)+1}{1-\ln(x)} \underset{BH}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to +\infty} \frac{\cancel{\frac{1}{x}}\,1}{-\cancel{\frac{1}{x}}\,1} = -1$$

g) $$\lim_{x\to +\infty} \frac{\ln(x^2)}{\ln^2(x)} \underset{BH}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to +\infty} \frac{\frac{2\cancel{x}}{x^{\cancel{2}}}}{2\ln(x)\cdot\frac{1}{x}} = \lim_{x\to +\infty} \frac{\cancel{2}}{\cancel{x}} \cdot \frac{\cancel{x}}{\cancel{2}\ln(x)}$$

$$= \frac{1}{+\infty} = 0$$

h) $$\lim_{x\to +\infty} \frac{\ln(x)}{e^x} \underset{BH}{\overset{\frac{+\infty}{+\infty}}{=}} \lim_{x\to +\infty} \frac{\frac{1}{x}}{e^x}$$

$$= \lim_{x\to +\infty} \frac{1}{x}\cdot\frac{1}{e^x} = \frac{1}{+\infty} = 0$$

$$\left[\ln^2(x)\right]' = \left[\left(\ln(x)\right)^2\right]'$$

$(u^2)' = 2u \cdot u'$ avec $u = \ln(x)$

$(u^n)' = n\,u^{n-1} \cdot u'$