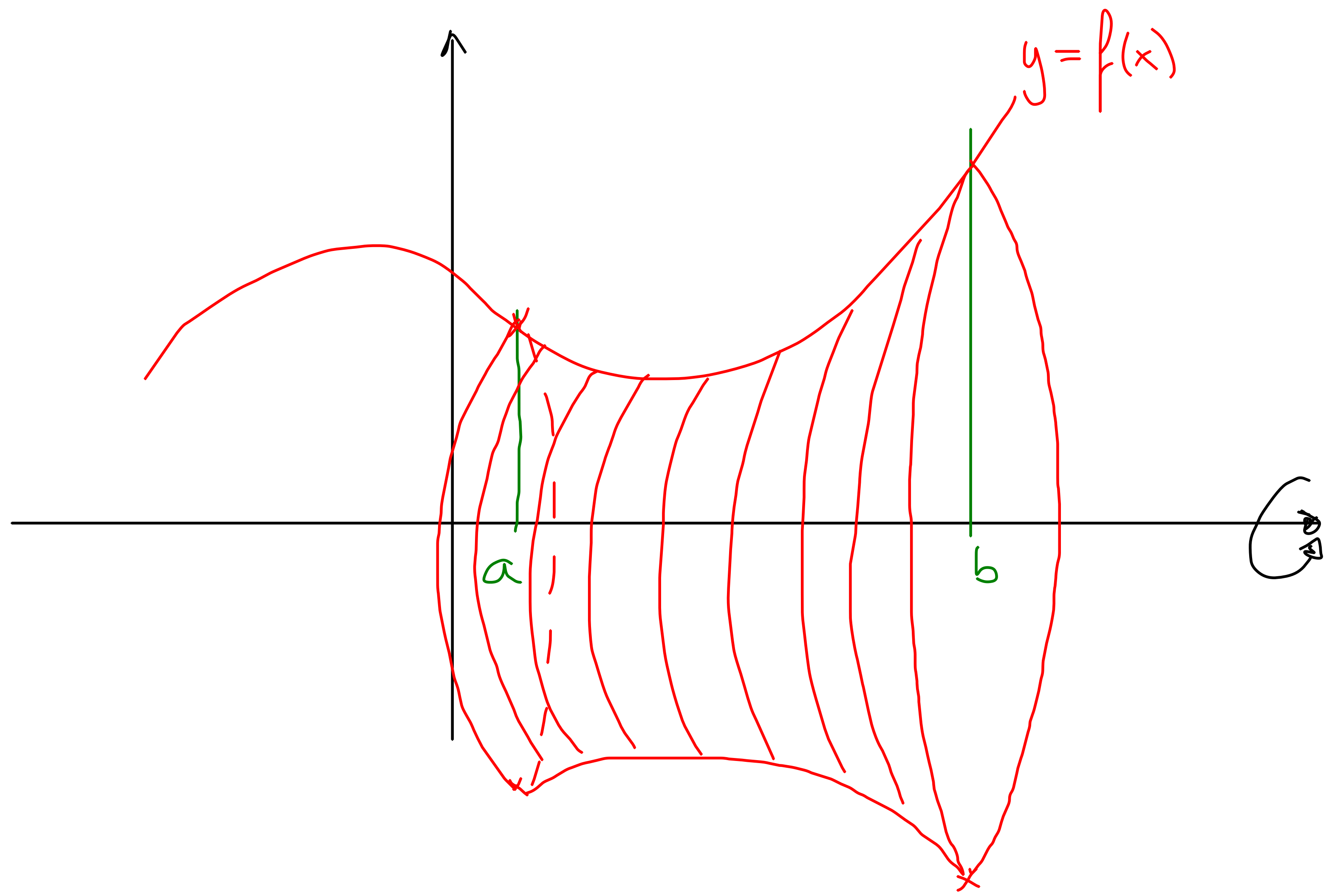
Volume d'un solide de révolution

Un solide de révolution est un solide obtenu par la rotation autour d'un axe d'un domaine borné.

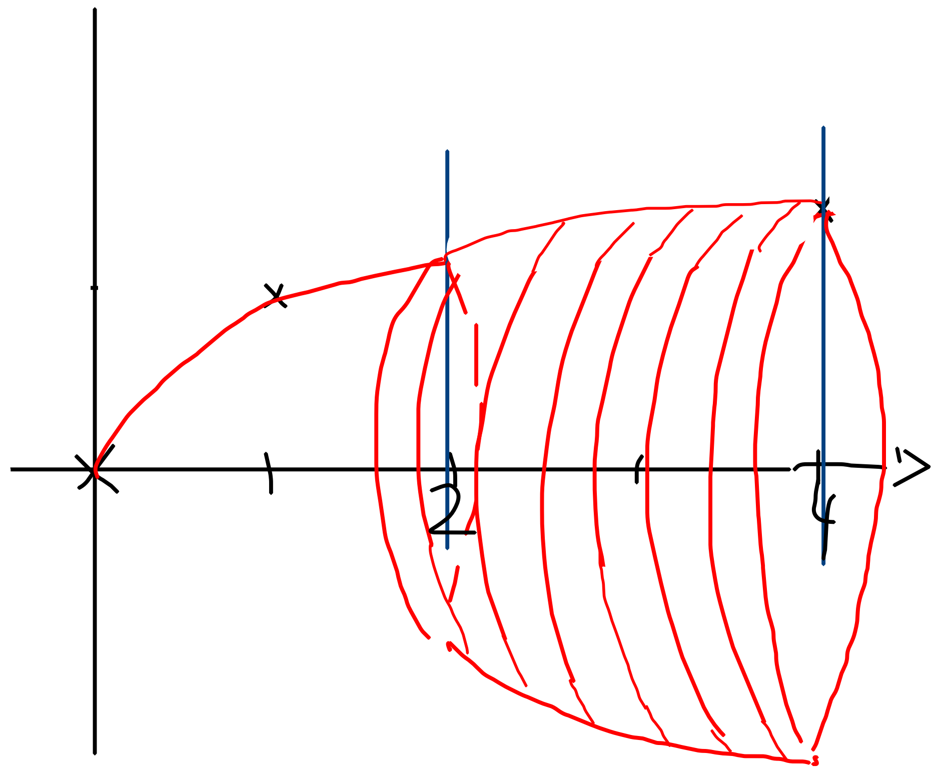


On fait tourner autour de l'axe Ox le domaine limité par $y = f(x)$, Ox, $x = a$ et $x = b$

$$\boxed{V = \pi \int_a^b f(x)^2 \, dx}$$

Exemple :

$f(x) = \sqrt{x}$ $\qquad ED(f) = \mathbb{R}_+$



Volume du solide engendré par la rotation autour de Ox du domaine limité par $y = f(x)$, Ox, $x = 2$ et $x = 4$

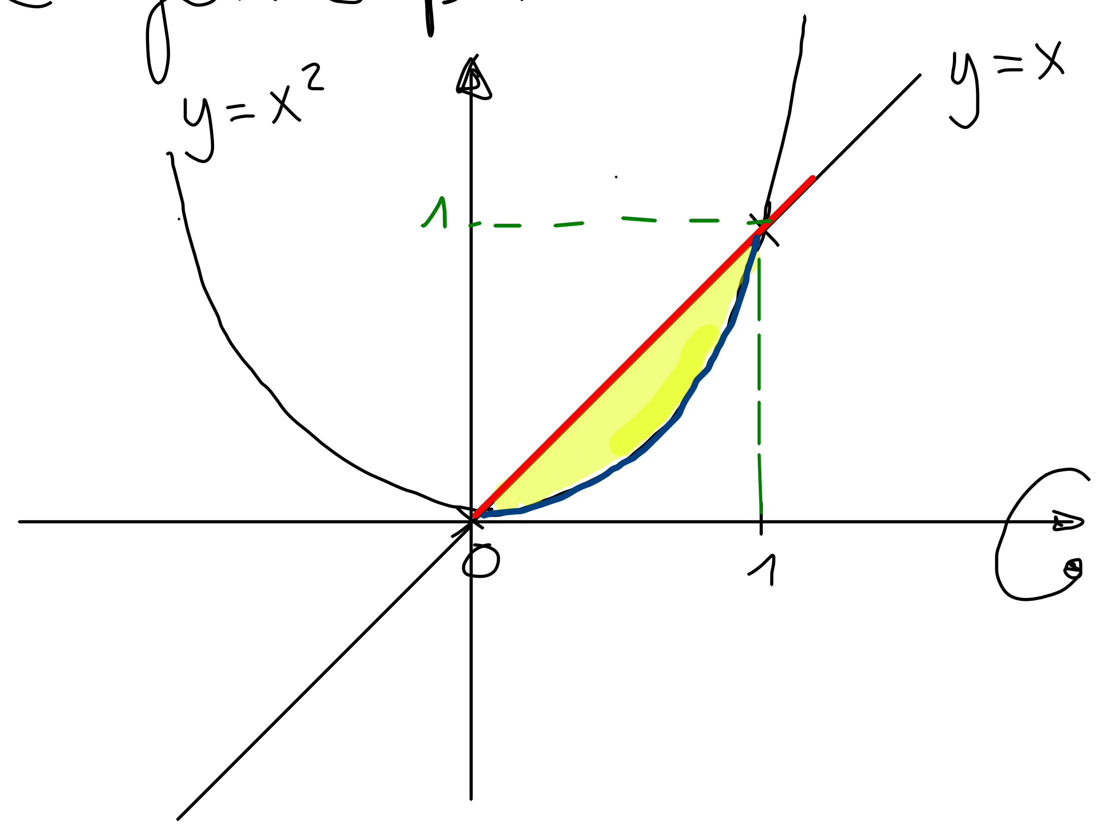
$$V = \pi \int_2^4 (\sqrt{x})^2 \, dx = \pi \int_2^4 x \, dx = \pi \cdot \frac{1}{2} x^2 \Big|_2^4$$

$$= \pi \left(\frac{1}{2} \cdot 16 - \frac{1}{2} \cdot 4 \right) = \pi (8 - 2) = 6\pi \ u^3$$

Exemple

Calculer le volume du solide engendré par la rotation autour de Ox du domaine fermé par :

a) $y = x$ et $y = x^2$



ptd'n : $x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$0 \to (0;0)$ $\quad 1 \to (1;1)$

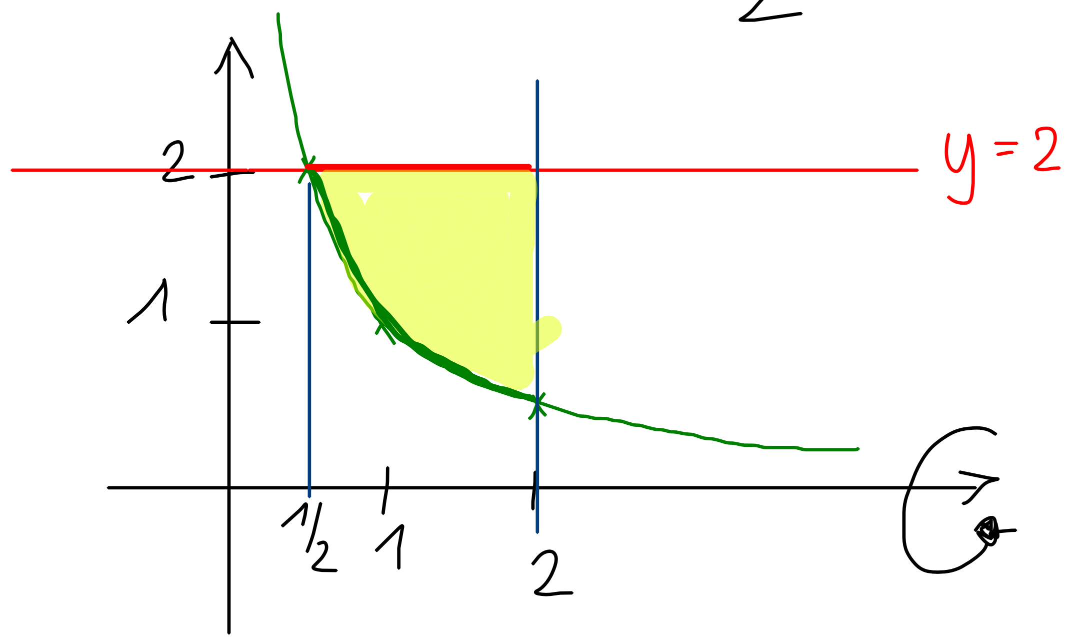
$$V_1 = \pi \int_0^1 x^2\,dx = \pi \cdot \frac{1}{3}x^3 \Big|_0^1 = \pi\left(\frac{1}{3} - 0\right) = \frac{\pi}{3}$$

$$V_2 = \pi \int_0^1 (x^2)^2\,dx = \pi \int_0^1 x^4\,dx = \pi\, \frac{1}{5}x^5 \Big|_0^1 = \pi\left(\frac{1}{5} - 0\right) = \frac{\pi}{5}$$

$$\Rightarrow \quad V = \left| V_1 - V_2 \right| = \left| \frac{\pi}{3} - \frac{\pi}{5} \right| = \pi \left| \frac{1}{3} - \frac{1}{5} \right| = \frac{2\pi}{15}\ u^3$$

b) par $y = \frac{1}{x}$, $y = 2$ et $x = 2$

pt d'∩ : $\frac{1}{x} = 2 \iff x = \frac{1}{2}$



$$V_1 = \pi \int_{1/2}^{2} \left(\frac{1}{x}\right)^2 dx = \pi \int_{1/2}^{2} \frac{1}{x^2} dx = \pi \int_{1/2}^{2} x^{-2} dx = \pi \cdot \frac{1}{-1} x^{-1} \Big|_{1/2}^{2}$$

$$= \pi \cdot \frac{-1}{x} \Big|_{1/2}^{2} = \pi \left(-\frac{1}{2} - (-2)\right) = \pi \left(-\frac{1}{2} + 2\right) = \frac{3\pi}{2}$$

$$V_2 = \pi \int_{1/2}^{2} 2^2 dx = \pi \cdot 4x \Big|_{1/2}^{2} = \pi (8 - 2) = 6\pi$$

$$\Rightarrow \quad V = \left| \frac{3\pi}{2} - 6\pi \right| = \pi \left| \frac{3}{2} - 6 \right| = \pi \left| -\frac{9}{2} \right| = \frac{9\pi}{2} \, u^3$$