Ex 1.1.7

a) $\int \frac{1}{x+1}\,dx = \underline{\ln(|x+1|) + C}$

b) $\int \frac{1}{3x+2}\,dx = \frac{1}{3}\int \frac{3}{3x+2}\,dx = \underline{\frac{1}{3}\ln(|3x+2|) + C} = \underline{\ln\left(\sqrt[3]{|3x+2|}\right) + C}$

$u = 3x+2$
$u' = 3$

c) $\int \frac{x-1}{x^2-2x+4}\,dx = \frac{1}{2}\int \frac{2(x-1)}{x^2-2x+4}\,dx = \underline{\frac{1}{2}\ln(|x^2-2x+4|) + C}$

$u = x^2-2x+4$
$u' = 2x-2 = 2(x-1)$

$= \underline{\ln\left(\sqrt{|x^2-2x+4|}\right) + C}$

d) $\int \left(x^2+x+1+\frac{3}{5x-1}\right)dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C_1 + \frac{3}{5}\int \frac{5}{5x-1}\,dx$

$= \underline{\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \frac{3}{5}\ln(|5x-1|) + C}$

e) $\int \frac{x^2+2x-2}{x-1}\,dx$

$\deg(N) \geq \deg(D) \Rightarrow$ div. euclidienne

$$\begin{array}{r|l} x^2+2x-2 & x-1 \\ -x^2+x & \overline{x+3} \\ \hline 3x-2 & \\ -3x+3 & \\ \hline 1 & \end{array}$$

$\Rightarrow f(x) = x+3+\frac{1}{x-1}$

$= \int \left(x+3+\frac{1}{x-1}\right)dx = \underline{\frac{1}{2}x^2 + 3x + \ln(|x-1|) + C}$

Ex 1.1.8

a) $$\int_2^5 \frac{dx}{x} = \ln(|x|)\Big|_2^5 = \ln(5)-\ln(2) = \underline{\ln\left(\frac{5}{2}\right)}$$

b) $$\int_{-1}^{-3} \frac{dx}{x} = \ln(|x|)\Big|_{-1}^{-3} = \ln(3)-\ln(1) = \underline{\ln(3)}$$

c) $$\int_{-1}^{4} \frac{dx}{x} = \lim_{\varepsilon\to 0_+}\left(\int_{-1}^{0-\varepsilon}\frac{dx}{x} + \int_{0+\varepsilon}^{4}\frac{dx}{x}\right) = \lim_{\varepsilon\to 0}\left(\ln(|x|)\Big|_{-1}^{0-\varepsilon} + \ln(|x|)\Big|_{0+\varepsilon}^{4}\right)$$

(⚠ pas défini en 0)

$$= \lim_{\varepsilon\to 0_+}\left(\ln(|0-\varepsilon|)-\ln(1)+\ln(4)-\ln(|0+\varepsilon|)\right)$$

$$= \underline{\ln(4)}$$

d) $$\int_1^4 \frac{dx}{2x+3} = \frac{1}{2}\int_1^4 \frac{2}{2x+3}\,dx = \frac{1}{2}\ln(|2x+3|)\Big|_1^4$$

$u = 2x+3$
$u' = 2$

$$= \frac{1}{2}\left(\ln(11)-\ln(5)\right) = \frac{1}{2}\ln\left(\frac{11}{5}\right) = \underline{\ln\left(\sqrt{\frac{11}{5}}\right)}$$

e) $$\int_2^6 \frac{8x^3+19x^2+15x+4}{x^2+2x+1}\,dx$$

$\deg(N) \geqslant \deg(D) \Rightarrow$ div. eucl.

$$\begin{array}{r|l} 8x^3+19x^2+15x+4 & x^2+2x+1 \\ -8x^3-16x^2-8x & 8x+3 \\ \hline 3x^2+7x+4 & \\ -3x^2-6x-3 & \\ \hline x+1 & \end{array}$$

$$= \int_2^6 \left(8x+3+\frac{x+1}{x^2+2x+1}\right)dx$$

$$= \int_2^6 (8x+3)\,dx + \frac{1}{2}\int_2^6 \frac{2(x+1)}{x^2+2x+1}\,dx$$

$$= 4x^2+3x\Big|_2^6 + \frac{1}{2}\ln(|x^2+2x+1|)\Big|_2^6 = 162-22+\frac{1}{2}\ln(49)-\frac{1}{2}\ln(9)$$

$$= 140+\ln(7)-\ln(3) = \underline{140+\ln\left(\frac{7}{3}\right)}$$