Ex 1.3.5

c) $\int (x+3)^3\,dx = \frac{1}{4}(x+3)^4 + C$

$u = x+3$
$u' = 1$

d) $\frac{1}{2}\int 2(2x-1)^2\,dx = \frac{1}{2}\cdot\frac{1}{3}(2x-1)^3 + C = \frac{1}{6}(2x-1)^3 + C$

$u = 2x-1$
$u' = 2$

e) $\frac{1}{7}\int 7(7x-2)^5\,dx = \frac{1}{7}\cdot\frac{1}{6}(7x-2)^6 + C = \frac{1}{42}(7x-2)^6 + C$

$u = 7x-2$
$u' = 7$

f) $\int (3x^2+x)^3(6x+1)\,dx = \frac{1}{4}(3x^2+x)^4 + C$

$u = 3x^2+x$
$u' = 6x+1$

g) $\frac{1}{8}\int (4x^2+3)^4\, x\cdot 8\,dx = \frac{1}{8}\cdot\frac{1}{5}(4x^2+3)^5 + C = \frac{1}{40}(4x^2+3)^5 + C$

$u = 4x^2+3$
$u' = 8x$

j) $\int \sqrt{x+3}\,dx = \int (x+3)^{1/2}\,dx = \frac{1}{3/2}(x+3)^{3/2} + C = \frac{2}{3}\sqrt{(x+3)^3} + C$

$u = x+3$
$u' = 1$

k) $\frac{1}{3}\int \frac{3\cdot 1}{\sqrt{3x+1}}\,dx = \frac{1}{3}\int 3(3x+1)^{-1/2}\,dx = \frac{1}{3}\cdot\frac{1}{1/2}(3x+1)^{1/2} + C = \frac{2}{3}\sqrt{3x+1} + C$

$u = 3x+1$
$u' = 3$

l) $\frac{1}{2}\int \frac{(x+1)\cdot 2}{\sqrt{x^2+2x}}\,dx = \frac{1}{2}\int (x^2+2x)^{-1/2}\cdot 2(x+1)\,dx = \frac{1}{2}\cdot\frac{1}{1/2}(x^2+2x)^{1/2} + C = \sqrt{x^2+2x} + C$

$u = x^2+2x$
$u' = 2x+2 = 2(x+1)$

Ex 1.3.6

a) $\int (3x^2-2x+3)\,dx = \underline{x^3 - x^2 + 3x + C}$

b) $\int \frac{3x^4-3x^2-7}{4x^2}\,dx$ deg(N) ⩾ deg(D) ⇒ division euclidienne

ici on n'est pas obligé de la poser car le dénominat. est un monôme, on peut diviser directement.

$= \int \left(\frac{3x^{\cancel{4}\,2}}{4\cancel{x^2}} - \frac{3\cancel{x^2}}{4\cancel{x^2}} - \frac{7}{4x^2}\right)dx$

⚠ $\frac{7}{4}\cdot\frac{1}{x^2} = \frac{7}{4}\cdot x^{-2}$ $(\neq 7\cdot 4x^{-2})$

$= \int \left(\frac{3x^2}{4} - \frac{3}{4} - \frac{7}{4}\cdot x^{-2}\right)dx$

$= \frac{3}{4}\cdot\frac{1}{3}x^3 - \frac{3}{4}x - \frac{7}{4}\cdot\frac{1}{-1}\; x^{-1} + C \quad = \underline{\frac{1}{4}x^3 - \frac{3}{4}x + \frac{7}{4x} + C}$

c) $\int 7\sqrt[4]{x^3}\,dx = \int 7x^{3/4}\,dx = 7\cdot\frac{1}{\frac{7}{4}}\,x^{7/4} + C = \underline{4\sqrt[4]{x^7} + C}$

d) $\int (\sqrt{x} - \sqrt[3]{x})\,dx = \int (x^{1/2} - x^{1/3})\,dx = \frac{1}{\frac{3}{2}}x^{3/2} - \frac{1}{\frac{4}{3}}x^{4/3} + C$

$= \underline{\frac{2}{3}\sqrt{x^3} - \frac{3}{4}\sqrt[3]{x^4} + C}$

i) $\int (3x^2-7)^2\,dx = \int (9x^4 - 42x^2 + 49)\,dx$

$= \frac{9}{5}x^5 - \frac{42}{3}x^3 + 49x + C = \underline{\frac{9}{5}x^5 - 14x^3 + 49x + C}$

⚠ ne pas confondre avec $\int \underbrace{(3x^2-7)^2}_{u^2}\cdot\underbrace{6x}_{u'}\,dx = \frac{1}{3}(3x^2-7)^3 + C$

j) $\int \sqrt{x}\cdot(x^2-5)\,dx = \int x^{\frac{1}{2}}(x^2-5)\,dx = \int \left(x^{\frac{1}{2}+2} - 5x^{\frac{1}{2}}\right)dx = \int \left(x^{\frac{5}{2}} - 5x^{\frac{1}{2}}\right)dx$

Un des facteurs n'est pas la dérivée interne de l'autre ⇒ on multiplie après transformation.

$= \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1} - 5\,\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + C$

$= \frac{2}{7}x^{\frac{7}{2}} - \frac{10}{3}x^{\frac{3}{2}} + C$

$= \underline{\frac{2}{7}\sqrt{x^7} - \frac{10}{3}\sqrt{x^3} + C}$

k) $\frac{1}{3}\int 3(3x-5)^6\,dx = \frac{1}{3}\cdot\frac{1}{7}(3x-5)^7 + C = \frac{1}{21}(3x-5)^7 + C$

l) $\int \frac{\overset{\overbrace{(-3)}^{u'}(-4)}{12}}{(4-3x)^4}\,dx = -4\int (-3)\cdot(4-3x)^{-4}\,dx = -4\cdot\frac{1}{-3}(4-3x)^{-3} + C = \frac{4}{3(4-3x)^3} + C$

$u = 4-3x$
$u' = -3$

m) $\int \sqrt[3]{(3x-8)^2}\,dx = \frac{1}{3}\int 3(3x-8)^{2/3}\,dx = \frac{1}{3}\cdot\frac{1}{\frac{5}{3}}(3x-8)^{\frac{5}{3}} + C$

$u = 3x-8$
$u' = 3$

$= \frac{1}{5}\sqrt[3]{(3x-8)^5} + C$

o) $\int x\sqrt{x^2+1}\,dx = \frac{1}{2}\int 2x\,(x^2+1)^{1/2}\,dx = \frac{1}{2}\cdot\frac{1}{\frac{3}{2}}(x^2+1)^{\frac{3}{2}} + C$

$u = x^2+1$
$u' = 2x$

$= \frac{1}{3}\sqrt{(x^2+1)^3} + C$

p) $\int \frac{2x-1}{\sqrt{x^2-x-1}}\,dx = \int (x^2-x-1)^{-\frac{1}{2}}(2x-1)\,dx = \frac{1}{\frac{1}{2}}(x^2-x-1)^{\frac{1}{2}} + C$

$u = x^2-x-1$
$u' = 2x-1$

$= 2\sqrt{x^2-x-1} + C$