Ex 1.3.16

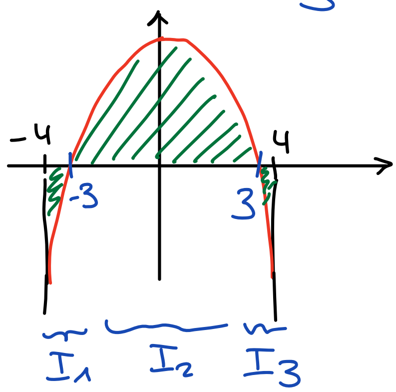
a) $f(x) = 9 - x^2$ (parabole $\cap$) $\qquad a = -4$ et $b = 4$

$ED(f) = \mathbb{R}$

zéros : $9 - x^2 = 0 \iff (3+x)(3-x) = 0 \iff x = \begin{cases} -3 \\ 3 \end{cases}$

signe :

x	-4		-3		3		4
$\operatorname{sgn}(f)$		$-$	0	$+$	0	$-$	
		I_1		I_2		I_3	



$$I_1 = \int_{-4}^{-3} (9 - x^2)\,dx = 9x - \frac{1}{3}x^3 \Big|_{-4}^{-3}$$

$$= (-27 + 9) - \left(-36 + \frac{64}{3}\right) = -\frac{10}{3}$$

$$I_2 = \int_{-3}^{3} (9 - x^2)\,dx = 9x - \frac{1}{3}x^3 \Big|_{-3}^{3}$$

$$= (27 - 9) - (-27 + 9) = 36$$

$$I_3 = \int_{3}^{4} (9 - x^2)\,dx = 9x - \frac{1}{3}x^3 \Big|_{3}^{4}$$

$$= \left(36 - \frac{64}{3}\right) - (27 - 9) = -\frac{10}{3}$$

$$\Rightarrow A = \left|-\frac{10}{3}\right| + 36 + \left|-\frac{10}{3}\right| = 36 + 2 \cdot \frac{10}{3} = \frac{128}{3}\ u^2$$

b) $f(x) = \frac{4}{x^2} - 1 \qquad a = 1 \;, b = 4$

$$f(x) = \frac{4-x^2}{x^2} = \frac{(2-x)(2+x)}{x^2}$$

← zéros : ± 2

← v.i. : 0 (2)

$ED(f) = \mathbb{R}^*$ signe :

x		-2		0	1	2	4	
$sgn(f)$	$-$	0	$+$	$\|$	$+$	0		$-$
				(2)	I_1	I_2		

$f(1000) : -$

$$I_1 = \int_1^2 \left(\frac{4}{x^2} - 1\right)dx = \int_1^2 \left(4x^{-2} - 1\right)dx = 4 \cdot \frac{1}{-1} x^{-1} - x \Big|_1^2$$

$$= -\frac{4}{x} - x \Big|_1^2 = (-2-2) - (-4-1) = 1$$

$$I_2 = \int_2^4 \left(\frac{4}{x^2} - 1\right)dx = -\frac{4}{x} - x \Big|_2^4 = (-1-4) - (-2-2) = -1$$

$$\Rightarrow \quad \mathcal{A} = 1 + |-1| = 1 + 1 = \underline{2u^2}$$

d) $f(x) = \sqrt{2x-4}$ $\qquad a = 2 \ , \ b = 10$

$ED(f) = [2 ; +\infty[$ $\quad$ car $\quad 2x-4 \geq 0$

x		2		10
	$-$	0	$+$	

$$\mathcal{A} = \int_2^{10} \sqrt{2x-4}\, dx = \int_2^{10} (2x-4)^{1/2} dx = \frac{1}{2}\int_2^{10} (2x-4)^{1/2} \cdot 2 dx$$

$u = 2x-4$
$u' = 2$

$$= \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (2x-4)^{3/2} \Big|_2^{10} = \frac{1}{3} \sqrt{(2x-4)^3} \Big|_2^{10}$$

$$= \frac{1}{3} 4^3 - \frac{1}{3} \cdot 0 = \underline{\frac{64}{3} u^2}$$

Ex 1.3.17

a) $f(x) = x^3 - 2x^2 - x + 2 = x^2(x-2) - 1(x-2) = (x-2)(x^2-1) = (x-2)(x+1)(x-1)$

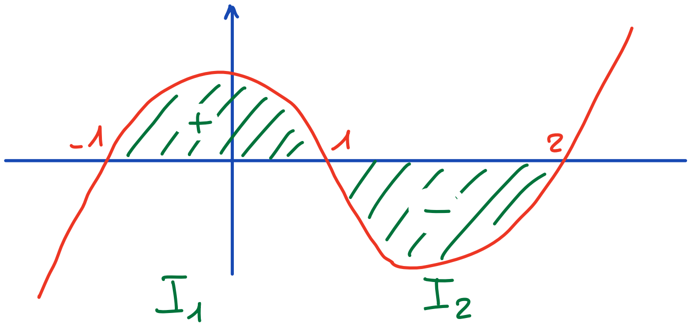
$ED(f) = \mathbb{R}$ zéros : -1, 1 et 2

signe :

x		-1		1		2	
$\text{sgn}(f)$	$-$	0	$+$	0	$-$	0	$+$

$f(1000)$: $+$

croquis :



$$I_1 = \int_{-1}^{1} (x^3 - 2x^2 - x + 2)\,dx = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x \Big|_{-1}^{1}$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) - \left(\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2\right) = \frac{8}{3}$$

$$I_2 = \int_{1}^{2} f(x)\,dx = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x \Big|_{1}^{2}$$

$$= \left(4 - \frac{16}{3} - 2 + 4\right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) = -\frac{5}{12}$$

$$\Rightarrow \quad \mathcal{A} = \frac{8}{3} + \frac{5}{12} = \frac{37}{12} \ u^2$$

b) $f(x) = x\sqrt{4-x^2}$ cond : $4-x^2 \geq 0 \iff (2+x)(2-x) \geq 0$

x		-2		2	
$4-x^2$	$-$	0	$+$	0	$-$

$ED(f) = [2;2]$

zéros : $-2; 0; 2$

signe :

x		-2		0		2	
$sgn(f)$	////	0	$-$	0	$+$	0	////

$\underbrace{\quad -\quad}_{I_1} \quad \underbrace{\quad +\quad}_{I_2}$

$$I_1 = \int_{-2}^{0} x\sqrt{4-x^2}\,dx = \int_{-2}^{0} x(4-x^2)^{1/2}dx = -\frac{1}{2}\int_{-2}^{0} -2x(4-x^2)^{1/2}dx$$

$u = 4-x^2$
$u' = -2x$

$$= -\frac{1}{2}\cdot\frac{1}{\frac{3}{2}}(4-x^2)^{3/2}\Big|_{-2}^{0} = -\frac{1}{3}\sqrt{(4-x^2)^3}\Big|_{-2}^{0}$$

$$= -\frac{1}{3}8 - \left(-\frac{1}{3}\cdot 0\right) = -\frac{8}{3}$$

$$I_1 = \int_{0}^{2} x\sqrt{4-x^2}\,dx = -\frac{1}{3}\sqrt{(4-x^2)^3}\Big|_{-2}^{0} = 0 - \left(-\frac{8}{3}\right) = \frac{8}{3}$$

$\Rightarrow \mathcal{A} = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}\ u^2$