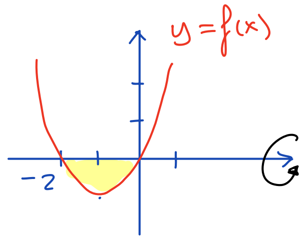
Ex 1.3.24



a) $f(x) = x^2+2x = x(x+2) \Rightarrow$ zéros : -2 et 0

$$V = \pi \int_{-2}^{0} (x^2+2x)^2\,dx = \pi \int_{-2}^{0} (x^4+4x^3+4x^2)\,dx$$

$$= \pi \left(\frac{1}{5}x^5 + x^4 + \frac{4}{3}x^3\right)\Big|_{-2}^{0} = \pi\left(0-\left(-\frac{32}{5}+16-\frac{32}{3}\right)\right) = \frac{16\pi}{15}\,u^3$$

b) $f(x) = \sqrt{1-x^2}$ zéros : $1-x^2=0 \Leftrightarrow (1+x)(1-x)=0 \Rightarrow -1$ et 1

$$V = \pi \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx = \pi \int_{-1}^{1} (1-x^2)\,dx = \pi\left(x-\frac{1}{3}x^3\right)\Big|_{-1}^{1}$$

$$= \pi\left(1-\frac{1}{3}-\left(-1+\frac{1}{3}\right)\right) = \pi\left(\frac{2}{3}-\left(-\frac{2}{3}\right)\right) = \frac{4\pi}{3}\,u^3$$

↑ Volume d'une sphère de rayon $=1$ $\left(V=\frac{4}{3}\pi r^3\right)$

$$y=f(x) \Leftrightarrow y=\sqrt{1-x^2}$$
$$\Leftrightarrow y^2 = 1-x^2$$
$$\Leftrightarrow \underbrace{x^2+y^2=1}_{\text{équation d'un cercle de rayon}=1}$$

Ex 1.3.26

a) $f(x) = x+1 \qquad a=1 \qquad b=3$

$$V = \pi \int_1^3 (x+1)^2 dx = \pi \int_1^3 (x^2+2x+1) dx = \pi \left(\frac{1}{3}x^3 + x^2 + x \right) \Big|_1^3$$

$$= \pi \left(9+9+3 - \left(\frac{1}{3}+1+1 \right) \right) = \pi \left(21 - \frac{7}{3} \right) = \frac{56\pi}{3} \ u^3$$

Variante: $\pi \int_1^3 (x+1)^2 dx = \pi \cdot \frac{1}{3}(x+1)^3 \Big|_1^3 = \pi \cdot \frac{1}{3}(64-8) = \frac{56\pi}{3} u^3$

$u = x+1$
$u' = 1$

b) $f(x) = x^2 \qquad a=0 \qquad b=4$

$$V = \pi \int_0^4 (x^2)^2 dx = \pi \int_0^4 x^4 dx = \pi \cdot \frac{1}{5}x^5 \Big|_0^4 = \pi \left(\frac{1024}{5} - 0 \right) = \frac{1024\pi}{5} u^3$$

c) $f(x) = \frac{1}{x+1} \qquad a=1 \qquad b=2$

$$V = \pi \int_1^2 \left(\frac{1}{x+1} \right)^2 dx = \pi \int_1^2 (x+1)^{-2} dx = \pi \frac{1}{-1} (x+1)^{-1} \Big|_1^2 = -\pi \cdot \frac{1}{x+1} \Big|_1^2$$

$u = x+1$
$u' = 1$

$$= -\pi \left(\frac{1}{3} - \frac{1}{2} \right) = -\pi \cdot \left(-\frac{1}{6} \right) = \frac{\pi}{6} \ u^3$$

Ex 1.3.28

a) $f(x) = \sqrt{x}$ $\quad g(x) = x^2$

pts d'∩ : $\sqrt{x} = x^2 \iff x = x^4 \iff x^4 - x = 0 \iff x(x^3-1) = 0$

$$\iff \underset{\substack{\downarrow \\ 0}}{x}\,\underset{\substack{\downarrow \\ 1}}{(x-1)}\underbrace{(x^2+x+1)}_{\Delta<0} = 0$$

$$V_1 = \pi\int_0^1 (\sqrt{x})^2\,dx = \pi\int_0^1 x\,dx = \pi.\ \frac{1}{2}x^2\Big|_0^1 = \pi\left(\frac{1}{2}-0\right) = \frac{\pi}{2}$$

$$V_2 = \pi\int_0^1 (x^2)^2\,dx = \pi\int_0^1 x^4\,dx = \pi.\ \frac{1}{5}x^5\Big|_0^1 = \pi\left(\frac{1}{5}-0\right) = \frac{\pi}{5}$$

$$V = |V_1 - V_2| = \left|\frac{\pi}{2}-\frac{\pi}{5}\right| = \pi\left|\frac{1}{2}-\frac{1}{5}\right| = \frac{3\pi}{10}\ u^3$$

b) $f(x) = x^2-2x+6$ $\quad g(x) = -x^2+10$

pts d'∩ : $x^2-2x+6 = -x^2+10 \iff 2x^2-2x-4 = 0$

$$\iff x^2-x-2 = 0 \iff \underset{\substack{\downarrow \\ 2}}{(x-2)}\underset{\substack{\downarrow \\ -1}}{(x+1)} = 0$$

$$V_1 = \pi\int_{-1}^{2} (x^2-2x+6)^2\,dx = \pi\int_{-1}^{2}\left(x^4+4x^2+36-4x^3+12x^2-24x\right)dx$$

$$= \pi\int_{-1}^{2}(x^4-4x^3+16x^2-24x+36)\,dx = \pi\left(\frac{1}{5}x^5 - x^4 + \frac{16}{3}x^3 - 12x^2 + 36x\right)\Bigg|_{-1}^{2}$$

$$= \pi\left(\left(\frac{32}{5} - 16 + \frac{128}{3} - 48 + 72\right) - \left(-\frac{1}{5} - 1 - \frac{16}{3} - 12 - 36\right)\right) = \pi\left(\frac{856}{15}+\frac{818}{15}\right) = \frac{558\pi}{5}$$

$$V_2 = \pi\int_{-1}^{2}(-x^2+10)^2\,dx = \pi\int_{-1}^{2}(x^4-20x^2+100)\,dx = \pi\left(\frac{1}{5}x^5 - \frac{20}{3}x^3 + 100x\right)\Bigg|_{-1}^{2}$$

$$= \pi\left(\left(\frac{32}{5} - \frac{160}{3} + 200\right) - \left(-\frac{1}{5} + \frac{20}{3} - 100\right)\right) = \pi\left(\frac{2296}{15} + \frac{1403}{15}\right) = \frac{1233\pi}{5}$$

$$\Rightarrow V = |V_1 - V_2| = \left|\frac{558\pi}{5} - \frac{1233\pi}{5}\right| = \pi\left|\frac{558}{5} - \frac{1233}{5}\right| = 135\,\pi$$