

Ex1

a) $\left(-\frac{1}{3}\right)^{-2} = (-3)^2 = \underline{9}$

b) $\frac{4^{-5}}{4^{-3}} = 4^{-5-(-3)} = 4^{-2} = \frac{1}{4^2} = \underline{\frac{1}{16}}$

Ex2

a) $((-5)^4)^{-3} = (5^4)^{-3} = 5^{-12} = \underline{\frac{1}{5^{12}}}$

b) $\left(\frac{3}{5}\right)^3 \div \left(\frac{3}{25}\right)^2 = \frac{3^3}{5^3} \cdot \frac{5^4}{3^2} = 3 \cdot 5 = \underline{15}$

c) $\frac{(2 \cdot 4 \cdot 16)^5}{2^{35}} = \frac{(2^{1+2+4})^5}{2^{35}} = \frac{2^{35}}{2^{35}} = \underline{1}$

d) $\left(\frac{a^{-3}}{a^5}\right)^{-3} = (a^{-8})^{-3} = \underline{a^{24}}$

e) $100 \cdot \frac{100'000}{10^{-4}} \cdot 10^{-15} = 10^2 \cdot \frac{10^5}{10^{-4}} \cdot 10^{-15} = 10^{2+5-(-4)-15}$
 $= 10^{-4} = \underline{\frac{1}{10^4}} = \underline{\frac{1}{10'000}} = \underline{0,0001}$

f) $(2x^2y^4z)^3 = \underline{8x^6y^{12}z^3}$

g) $\left(\frac{2a}{b}\right)^3 \cdot \left(\frac{b}{a^3}\right)^2 = \frac{8a^3}{b^3} \cdot \frac{b^2}{a^6} = \underline{\frac{8}{a^3b}}$

h) $\frac{(27x^5y^3)^2}{(3xy)^3} \div \frac{x^6}{(y^2)^2} = \frac{3^6 x^{10} y^6}{3^3 x^3 y^3} \cdot \frac{y^4}{x^6} = 3^3 x y^7 = \underline{27xy^7}$

Ex 3

$$a) \left(-\frac{1}{3}\right)^{-3} = (-3)^3 = \underline{-27}$$

$$b) (-32^{1/5} + 32^{-1/5})^2 = \left(-2 + \frac{1}{32^{1/5}}\right)^2 = \left(-2 + \frac{1}{2}\right)^2 = \left(-\frac{3}{2}\right)^2 = \underline{\frac{9}{4}}$$

$$c) (-8)^{4/3} = (-2^3)^{4/3} = (-2)^{3 \cdot 4/3} = (-2)^4 = \underline{16}$$

$$d) \sqrt[3]{\sqrt[3]{3^{36}}} = \sqrt{3^{12}} = 3^6 = \underline{729}$$

Ex 4

$$a) \frac{2}{\sqrt{5}} = \underline{2 \cdot 5^{-1/2}}$$

$$b) 2\sqrt[5]{5} = \underline{2 \cdot 5^{1/5}}$$

$$c) \sqrt{\frac{2}{5}} = \left(\frac{2}{5}\right)^{1/2} = \underline{2^{1/2} \cdot 5^{-1/2}}$$

Ex 5

$$a) 2\sqrt{75} - 6\sqrt{12} = 2 \cdot 5\sqrt{3} - 6 \cdot 2\sqrt{3} = 10\sqrt{3} - 12\sqrt{3} = \underline{-2\sqrt{3}}$$

$$b) \frac{\sqrt{60} - 3}{\sqrt{15}} = \frac{2\sqrt{15} - 3}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{2 \cdot 15 - 3\sqrt{15}}{15}$$
$$= \frac{30 - 3\sqrt{15}}{15} = \underline{\frac{10 - \sqrt{15}}{5}}$$

$$\begin{aligned} \text{c) } (\sqrt{8}-5)^2 &= 8 - 2 \cdot 5\sqrt{8} + 25 \\ &= 33 - 2 \cdot 5 \cdot 2\sqrt{2} = \underline{33 - 20\sqrt{2}} \end{aligned}$$

$$\text{d) } \frac{3}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{3\sqrt{5}+6}{5-4} = \underline{3\sqrt{5}+6}$$

Ex 6

$$\text{a) } \left(\frac{a}{2}\right)^{-3} : a^{-2} = \left(\frac{2}{a}\right)^3 : \frac{1}{a^2} = \frac{2^3}{a^3} \cdot a^2 = \underline{\frac{8}{a}}$$

$$\text{b) } \frac{(a^{-3}b)^{-1}}{(a^2b^{-2})^{-3}} = \frac{a^3b^{-1}}{a^{-6}b^6} = a^{3-(-6)} b^{-1-6} = a^9 b^{-7} = \underline{\frac{a^9}{b^7}}$$

$$\begin{aligned} \text{c) } \sqrt[5]{a^3 \sqrt[3]{a}} &= (a^3 \cdot a^{1/3})^{1/5} = (a^{3+1/3})^{1/5} = (a^{10/3})^{1/5} \\ &= a^{\frac{10}{3} \cdot \frac{1}{5}} = a^{\frac{2}{3}} = \underline{\sqrt[3]{a^2}} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{(\sqrt{a})^3}{a \cdot \sqrt[3]{a^2}} &= \frac{a^{3/2}}{a \cdot a^{2/3}} = a^{3/2-1-2/3} = a^{\frac{9-6-4}{6}} \\ &= a^{-1/6} = \underline{\frac{1}{\sqrt[6]{a}}} \end{aligned}$$

Ex 7

$$\text{a) } 25^{x+2} = 125^x$$

$$(5^2)^{x+2} = (5^3)^x$$

$$5^{2x+4} = 5^{3x}$$

$$2x+4 = 3x$$

$$x = 4$$

$$\text{verif: } 25^6 = 5^{12} = 125^4 \quad \checkmark$$

$$\Rightarrow \underline{S = \{4\}}$$

$$b) (3^{x-3})^3 = \frac{3^{x+2}}{3^2}$$

$$3^{3x-9} = 3^x$$

$$3x-9 = x$$

$$2x = 9$$

$$x = \frac{9}{2}$$

$$\Rightarrow \underline{S = \left\{ \frac{9}{2} \right\}}$$

$$c) 16 \cdot 4^{3x+5} - 2^x = 0$$

$$2^4 \cdot (2^2)^{3x+5} = 2^x$$

$$2^{4+6x+10} = 2^x$$

$$6x+14 = x$$

$$5x = -14$$

$$x = -\frac{14}{5}$$

$$\Rightarrow \underline{S = \left\{ -\frac{14}{5} \right\}}$$

$$d) 16^x - 20 \cdot 4^x + 64 = 0$$

$$4^{2x} - 20 \cdot 4^x + 64 = 0$$

3 termes

chgmt de variable

$$y = 4^x$$

$$y^2 - 20y + 64 = 0$$

$$(y-16)(y-4) = 0$$

$$y = 4 \Rightarrow 4^x = 4 \Leftrightarrow x = 1$$

$$y = 16 \Rightarrow 4^x = 16 \Leftrightarrow x = 2$$

$$\Rightarrow \underline{S = \{1, 2\}}$$

Ex 8

$$a) \log_3(81) = \underline{4}$$

$$b) \log_4\left(\frac{1}{64}\right) = \underline{-3}$$

$$\Leftrightarrow 4^{-3} = \frac{1}{64} = \frac{1}{4^3}$$

$$c) \log_a(\sqrt{a}) = \underline{\frac{1}{2}}$$

$$d) \log(\sqrt[3]{100}) = \underline{\frac{2}{3}}$$

$$\Leftrightarrow 10^{2/3} = \sqrt[3]{10^2}$$