

Ex 2.1.1

a) $3 + (xz + y^2) = \underline{\underline{3+xz+y^2}}$

b) $3 - (xz + y^2) = \underline{\underline{3-xz-y^2}}$

c) $3(xz + y^2) = \underline{\underline{3xz+3y^2}}$

d) $(2a + b - c) + (3a - b + c) = \underline{\underline{2a+b-c+3a-b+c}} = \underline{\underline{5a}}$

e) $(2a + b - c) - (3a - b + c) = \underline{\underline{2a+b-c-3a+b-c}} = \underline{\underline{-a+2b-2c}}$

f) $(2a + b - c)(3a - b + c) = \underline{\underline{6a^2-2ab+2ac+3ab-b^2+bc-3ac+bc-c^2}} \\ = \underline{\underline{6a^2-b^2-c^2+ab-ac+2bc}}$

g) $(x^3 - 2x^2 - 5) + (-4x^3 - 1) = \underline{\underline{x^3-2x^2-5-4x^3-1}} = \underline{\underline{-3x^3-2x^2-6}}$

h) $(x^3 - 2x^2 - 5) - (-4x^3 - 1) = \underline{\underline{x^3-2x^2-5+4x^3+1}} = \underline{\underline{5x^3-2x^2-4}}$

i) $(x^3 - 2x^2 - 5)(-4x^3 - 1) = \underline{\underline{-4x^6-x^3+8x^5+2x^2+20x^3+5}} \\ = \underline{\underline{-4x^6+8x^5+19x^3+2x^2+5}}$

Ex 2.1.2

a) $(a + b)^2 = \underline{\underline{a^2+2ab+b^2}}$

b) $(a - b)^2 = \underline{\underline{a^2-2ab+b^2}}$

c) $(a + b)(a - b) = \underline{\underline{a^2-b^2}}$

d) $(a + b)^3 = \underline{\underline{a^3+3a^2b+3ab^2+b^3}}$

e) $(a - b)^3 = \underline{\underline{a^3-3a^2b+3ab^2-b^3}}$

f) $(a - b)(a^2 + ab + b^2) = \underline{\underline{a^3-b^3}}$

g) $(a + b)(a^2 - ab + b^2) = \underline{\underline{a^3+b^3}}$



Ex 2.1.3

$$a) (a+8)^2 = \underline{\underline{a^2 + 16a + 64}}$$

$$b) (y^4 - 3b)^3 = \underline{\underline{y^{12} - 3y^8 \cdot 3b + 3y^4 \cdot 9b^2 + 27b^3}} \\ = \underline{\underline{y^{12} - 9y^8 b + 27y^4 b^2 + 27b^3}}$$

$$c) (u-3)(u+3) = \underline{\underline{u^2 - 9}}$$

$$d) (2m - 5n)(4m^2 + 10mn + 25n^2) = (2m)^3 - (5n)^3 = \underline{\underline{8m^3 - 125n^3}}$$

$$e) (7-f)^2 = \underline{\underline{49 - 14f + f^2}}$$

$$f) (4 + 2z^2)^3 = \underline{\underline{64 + 3 \cdot 16 \cdot 2z^2 + 3 \cdot 4 \cdot 4z^4 + 8z^6}} \\ = \underline{\underline{64 + 96z^2 + 48z^4 + 8z^6}}$$

$$g) (3+y^3)(y^6 - 3y^3 + 9) = \underline{\underline{y^9 + 27}}$$

$$h) (x^2 + y^2)(x^2 - y^2) = \underline{\underline{x^4 - y^4}}$$

$$i) (t + 3u^5)^3 = \underline{\underline{t^3 + 3t^2 \cdot 3u^5 + 3t \cdot 9u^{10} + 27u^{15}}} \\ = \underline{\underline{t^3 + 9t^2 u^5 + 27tu^{10} + 27u^{15}}}$$

$$j) (2x-7)^2 = \underline{\underline{4x^2 - 28x + 49}}$$

$$k) (b^2 - c^3)(b^2c^3 + b^4 + c^6) = \underline{\underline{b^6 - c^9}}$$

$$l) (a-3b)^3 = \underline{\underline{a^3 - 3a^2 \cdot 3b + 3a \cdot 9b^2 + 27b^3}} \\ = \underline{\underline{a^3 - 9a^2 b + 27ab^2 + 27b^3}}$$

Ex 2.1.4

$$a) (x-1)^2 - (y+1)^2 = \underline{\underline{x^2 - 2x + 1 - (y^2 - 2y + 1)}} \\ = \underline{\underline{x^2 - 2x + 1 - y^2 + 2y - 1}} \\ = \underline{\underline{x^2 - y^2 - 2x + 2y}}$$

$$b) (1+x)^2 - (1-x)^2 = \underline{\underline{1+2x+x^2 - (1-2x+x^2)}} \\ = \underline{\underline{1+2x+x^2 - 1+2x-x^2}} = \underline{\underline{4x}}$$

$$c) \left(\frac{1}{2}x + \frac{1}{2}y\right)^2 - \left(\frac{1}{2}x - \frac{1}{2}y\right)^2 = \frac{1}{4}x^2 + \frac{1}{2}xy + \frac{1}{4}y^2 - \left(\frac{1}{4}x^2 - \frac{1}{2}xy + \frac{1}{4}y^2\right) = \underline{\underline{xy}}$$

$$d) (2x+y)^2 + (2x-y)^2 - 2(2x+y)(2x-y)$$
$$= 4x^2 + 4xy + y^2 + 4x^2 - 4xy + y^2 - 2(4x^2 - y^2)$$
$$= 8x^2 + 2y^2 - 8x^2 + 2y^2 = \underline{\underline{4y^2}}$$

$$e) (3x+y)(3x-y) - (3x+2y)^2 - (x-3y)^2$$
$$= 9x^2 - y^2 - (9x^2 + 12xy + 4y^2) - (x^2 - 6xy + 9y^2)$$
$$= \underline{\underline{9x^2 - y^2}} - \underline{\underline{9x^2 + 12xy + 4y^2}} - \underline{\underline{x^2 - 6xy + 9y^2}}$$
$$= \underline{\underline{-x^2 - 14y^2 - 6xy}}$$

Ex 2.1.6

$$p(x) = 2x^3 - 3x^2 + 5x - 1$$

$$q(x) = 3x^3 + 2x^2 - 4x + 2$$

a) $p(x) + q(x) = \underline{5x^3 - x^2 + x + 1}$

b) degré de $p \cdot q$: $2x^3 \cdot 3x^3 = 6x^6$ le degré vaut 6

terme de degré 4 : $2x^3 \cdot (-4x) + (-3x^2) \cdot 2x^2 + 5x \cdot 3x^3$

$$= -8x^4 - 6x^4 + 15x^4$$

$$= \underline{1}x^4 = \underline{1} \cdot x^4 \Rightarrow \text{le coefficient vaut } \underline{1}$$

Ex 2.1.7

$$p(x) = x^2 + x + 2$$

$$q(x) = x^3 - 2x$$

$$p(x) + q(x) = \underline{x^3 + x^2 - x + 2}$$

$$p(x) - q(x) = x^2 + x + 2 - x^3 + 2x = \underline{-x^3 + x^2 + 3x + 2}$$

$$\begin{aligned} p(x) \cdot q(x) &= (x^2 + x + 2)(x^3 - 2x) = x^5 - \underline{2x^3} + x^4 - 2x^2 + \underline{2x^3} - 4x \\ &= \underline{x^5 + x^4 - 2x^2 - 4x} \end{aligned}$$

Ex 2.1.8

$$a(x) = 3x^2 - 4x + 3$$

$$p(x) = x^4 + 2x^3 - 2x^2 - 4x + 17 \quad \text{et} \quad q(x) = 2x^3 - 3x^2 - 5x + 18$$

a) $(a(x))^2 = 9x^4 + \underline{16x^2} + 9 - 24x^3 + \underline{18x^2} - 24x$
 $= \underline{9x^4 - 24x^3 + 34x^2 - 24x + 9}$

b) $p(x) - q(x) = x^4 + \underline{2x^3} - \underline{2x^2} - \underline{4x} + \underline{17} - \underline{2x^3} + \underline{3x^2} + \underline{5x} - \underline{18}$
 $= \underline{x^4 + x^2 + x - 1}$

c) degré de $p(x) \cdot q(x)$: $x^4 \cdot 2x^3 = 2x^7 \Rightarrow \text{le degré vaut } \underline{7}$

d) coefficient du terme de degré 7 : 2

e) terme de degré 4 : $x^4 \cdot 18 + 2x^3 \cdot (-5x) - 2x^2 \cdot (-3x^2) - 4x \cdot 2x^3$

$$= 18x^4 - 10x^4 + 6x^4 - 8x^4$$

$$= 6x^4 \quad \Rightarrow \text{le coefficient vaut } \underline{\underline{6}}$$

Ex 2.1.9

a) $(2x-y-z) - (3x+2y-3z) - (4x+y-z) + (5x+4y-4z)$

$$= \underline{2x-y-z} - \underline{3x+2y-3z} - \underline{4x+y-z} + \underline{5x+4y-4z} = \underline{\underline{-z}}$$

f) $(3x^2-x+2)(4x+3)(2x-1)$

$$= (12x^3 + 9x^2 - 4x^2 - 3x + 8x + 6)(2x-1)$$

$$= (12x^3 + 5x^2 + 5x + 6)(2x-1)$$

$$= 24x^4 - 12x^3 + 10x^3 - 5x^2 + 10x^2 - 5x + 12x - 6$$

$$= \underline{\underline{24x^4 - 2x^3 + 5x^2 + 7x - 6}}$$

h) $x(x+1) - 3x(-x+3) + 2(x^2-x)$

$$= x^2 + x + 3x^2 - 9x + 2x^2 - 2x$$

$$= \underline{\underline{6x^2 - 10x}}$$

Ex 2.1.10

b) $(x+1)(x-1)^2 - (x-2)^3$ $(x-2)^3 = x^3 - 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 4 - 8$

$$\begin{aligned} &= (x+1)(x^2 - 2x + 1) - (x^3 - 6x^2 + 12x - 8) \\ &= \underline{x^3 - 2x^2 + x + x^2 - 2x + 1} - \underline{x^3 + 6x^2 - 12x + 8} \\ &= \underline{\underline{5x^2 - 13x + 9}} \end{aligned}$$

d) $(x+y)^3 - (x-y)^3 - (x^3 - y^3) - (x-y)(x^2 + xy + y^2)$

$$\begin{aligned} &= x^3 + 3x^2y + 3xy^2 + y^3 - (x^3 - 3x^2y + 3xy^2 - y^3) - x^3 + y^3 - (x^3 - y^3) \\ &= \underline{x^3 + 3x^2y + 3xy^2 + y^3} - \underline{x^3 + 3x^2y - 3xy^2 + y^3} - \underline{x^3 + y^3} - \underline{x^3 + y^3} \\ &= \underline{\underline{-2x^3 + 6x^2y + 4y^3}} \end{aligned}$$

g) $(2x-3y)^3 - 3y(x-3y)^2 - 9xy(4y-x)$

$$\begin{aligned} &= 8x^3 - 36x^2y + 54xy^2 + 27y^3 - 3y(x^2 - 6xy + 9y^2) - 36xy^2 + 9x^2y \\ &= \underline{8x^3} - \underline{36x^2y} + \underline{54xy^2} - \underline{27y^3} - \underline{3x^2y} + \underline{18xy^2} - \underline{27y^3} - \underline{36xy^2} + \underline{9x^2y} \\ &= \underline{\underline{8x^3 - 30x^2y + 36xy^2 - 54y^3}} \end{aligned}$$