Ex 2.5.2

a) $2x^2-7x-4$

résoudre : $2x^2-7x-4=0$ $\quad \Delta = 49-4\cdot 2\cdot(-4) = 81$

$$x_{1,2} = \frac{7\pm 9}{4} = \begin{cases} 4 \\ -1/2 \end{cases} \Rightarrow S = \left\{-\frac{1}{2}\,;\,4\right\}$$

factoriser : $2x^2-7x-4 = 2\left(x-4\right)\left(x+\frac{1}{2}\right) = (x-4)(2x+1)$

c) $6x^2-25x-25$

résoudre : $6x^2-25x-25=0$ $\quad \Delta = 25^2-4\cdot 6\cdot(-25) = 1225 = 35^2$

$$x_{1,2} = \frac{25\pm 35}{12} = \begin{cases} 5 \\ -\frac{10}{12} = -\frac{5}{6} \end{cases} \Rightarrow S = \left\{-\frac{5}{6}\,;\,5\right\}$$

factoriser : $6x^2-25x-25 = 6\left(x-5\right)\left(x+\frac{5}{6}\right) = (x-5)(6x+5)$

d) $6x^2-20x+20$

résoudre : $6x^2-20x+20=0$ $\quad \Delta = 20^2-4\cdot 6\cdot 20 = -80 < 0$

factoriser : $6x^2-20x+20$ n'est pas factorisable $\Rightarrow S = \emptyset$

Ex 2.5.3

a) $x^2 - 9 = 0$

PR $(x+3)(x-3) = 0$

$x+3=0 \quad x=-3$; $x-3=0 \quad x=3$

$\Rightarrow S = \{\pm 3\}$

b) $4x^2 - 1 = 0$

PR $(2x+1)(2x-1) = 0$

$2x+1=0 \quad 2x=-1 \quad x=-\frac{1}{2}$; $\vdots \quad x=\frac{1}{2}$

$\Rightarrow S = \{\pm \frac{1}{2}\}$

c) $(x-2)^2 - 9(x-2) = 0$

GR $(x-2)[(x-2)-9] = 0$

$(x-2)(x-11) = 0$

$2 \quad 11$

$\Rightarrow S = \{2; 11\}$

d) $(x^2-x-6)(x+5) = 0$

SP $(x+2)(x-3)(x+5) = 0$

$-2 \quad 3 \quad -5$

$\Rightarrow S = \{-5; -2; 3\}$

e) $x^4 - 5x^2 + 4 = 0$

$y = x^2 \Rightarrow y^2 - 5y + 4 = 0$

SP $(y-4)(y-1) = 0$

$\Rightarrow (x^2-4)(x^2-1) = 0$

PR $(x+2)(x-2)(x+1)(x-1) = 0$

$-2 \quad 2 \quad -1 \quad 1$

$\Rightarrow S = \{\pm 1; \pm 2\}$

f) $(x-1)(x^2+1) = 0$

$x-1$: 1 ; x^2+1 : pas factorisable, pas de zéro

$\Rightarrow$ $S = \{1\}$

g) $x^3+x^2 = 4x+4$

GR $x^3+x^2-4x-4 = 0$

$x^2(x+1) - 4(x+1) = 0$

PR $(x+1)(x^2-4) = 0$

$(x+1)(x+2)(x-2) = 0$

-1 ; -2 ; 2

$\Rightarrow$ $S = \{-1; -2; 2\}$

h) $x^2-9-4(x-3) = 0$

$(x+3)(x-3) - 4(x-3) = 0$

GR $(x-3)[(x+3)-4] = 0$

$(x-3)(x-1) = 0$

3 ; 1

$\Rightarrow$ $S = \{1; 3\}$

j) $x^3+2x^2-x-2 = 0$

GR $x^2(x+2) - (x+2) = 0$

PR $(x+2)(x^2-1) = 0$

$(x+2)(x+1)(x-1) = 0$

-2 ; -1 ; 1

$\Rightarrow$ $S = \{-2; -1; 1\}$

Ex 2.5.4

a) $(x^2-8x+12)(x+2)^3 = 0$

SP $(x-6)(x-2)(x+2)^3 = 0$

$6 \quad 2 \quad -2$ (valeur triple)

$\Rightarrow S = \{-2; 2; 6\}$

b) $(x-3)(x^2-4) = 0$

PR $(x-3)(x+2)(x-2) = 0$

$3 \quad -2 \quad 2$

$\Rightarrow S = \{-2; 2; 3\}$

c) $x^3+2x^2-4x = 8$

$x^3+2x^2-4x-8 = 0$

GR $x^2(x+2) - 4(x+2) = 0$

$(x+2)(x^2-4) = 0$

$(x+2)(x+2)(x-2) = 0$

$(x+2)^2(x-2) = 0$

-2 (double) $\quad 2$

$\Rightarrow S = \{\pm 2\}$

d) $(2x^2+3x+1)^2 - (2x^2-4x-1)^2 = 0$

PR $[(2x^2+3x+1)+(2x^2-4x-1)][(2x^2+3x+1)-(2x^2-4x-1)] = 0$

$(4x^2-x)(2x^2+3x+1-2x^2+4x+1)$

MEE $x(4x-1)(7x+2) = 0$

$0 \quad 1/4 \quad -2/7$

$\Rightarrow S = \{0; \frac{1}{4}; -\frac{2}{7}\}$

e) $x(x-2)+(x-3)(x-2)=0$

MEE $(x-2)[x+(x-3)]=0$

$(x-2)(2x-3)=0$

$x=2$; $x=3/2$

$\Rightarrow S=\{2;\frac{3}{2}\}$

f) $6x^2=3x^3-72x$

$-3x^3+6x^2+72x=0$

$3x^3-6x^2-72x=0$

MEE $3x(x^2-2x-24)=0$

SP $3x(x-6)(x+4)=0$

$x=0$; $x=6$; $x=-4$

$\Rightarrow S=\{0;6;-4\}$

g) $x^3+3x^2=9x+27$

$x^3+3x^2-9x-27=0$

GR $x^2(x+3)-9(x+3)=0$

$(x+3)(x^2-9)=0$

PR $(x+3)(x+3)(x-3)=0$

$(x+3)^2(x-3)=0$

$x=-3$ (double) ; $x=3$

$\Rightarrow S=\{\pm 3\}$