## Ex 4.2.1

**a)** $7^{x+6} = 7^{3x+4}$ $\iff$ $x+6 = 3x+4$

$\iff$ $-2x = -2$

$\iff$ $x = 1$ $\implies$ $\underline{S = \{1\}}$

**b)** $6^{7-x} = 6^{2x+1}$ $\iff$ $7-x = 2x+1$

$\iff$ $-3x = -6$

$\iff$ $x = 2$ $\implies$ $\underline{S = \{2\}}$

**c)** $3^{2x+3} = 3^{x^2}$ $\iff$ $2x+3 = x^2$

$\iff$ $x^2 - 2x - 3 = 0$

$\iff$ $(x-3)(x+1) = 0$

$\iff$ $x = 3$ ou $x = -1$ $\implies$ $\underline{S = \{-1; 3\}}$

**d)** $9^{x^2} = 3^{3x+2}$ $\iff$ $(3^2)^{x^2} = 3^{3x+2}$

$\iff$ $3^{2x^2} = 3^{3x+2}$

$\iff$ $2x^2 = 3x+2$

$\iff$ $2x^2 - 3x - 2 = 0$ $\qquad \Delta = 9+16 = 25$

$\iff$ $x_{1,2} = \dfrac{3 \pm 5}{4} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$ $\implies$ $\underline{S = \{-\frac{1}{2}; 2\}}$

**e)** $2^{-100x} = 0.5^{x-4}$ $\iff$ $2^{-100x} = \left(\dfrac{1}{2}\right)^{x-4}$

$\iff$ $2^{-100x} = (2^{-1})^{x-4}$

$\iff$ $2^{-100x} = 2^{-x+4}$

$\iff$ $-100x = -x+4$

$\iff$ $-99x = 4$

$\iff$ $x = -\dfrac{4}{99}$ $\implies$ $\underline{S = \left\{-\dfrac{4}{99}\right\}}$

**f)** $\left(\dfrac{1}{4}\right)^{6-x} = 4 \iff (4^{-1})^{6-x} = 4$

$\iff 4^{-6+x} = 4$

$\iff -6+x = 1$

$\iff x = 7 \implies \underline{S = \{7\}}$

**g)** $27^{x-1} = 9^{2x-3} \iff (3^3)^{x-1} = (3^2)^{2x-3}$

$\iff 3^{3x-3} = 3^{4x-6}$

$\iff 3x-3 = 4x-6$

$\iff -x = -3 \implies \underline{S = \{3\}}$

**h)** $2^x \cdot 4^x = -5 \iff 2^x \cdot 2^{2x} = -5$

$\iff \underbrace{2^{3x}}_{>0} = -5 < 0 \text{ impossible} \implies \underline{S = \emptyset}$

**i)** $(5^{x-2})^4 = 125 \cdot 5^{5x-3} \iff 5^{4x-8} = 5^3 \cdot 5^{5x-3}$

$\iff 5^{4x-8} = 5^{5x}$

$\iff 4x-8 = 5x$

$\iff -x = 8 \implies \underline{S = \{-8\}}$

**j)** $(3^{x-1})^3 = 9 \cdot 3^{x-2} \iff 3^{3x-3} = 3^2 \cdot 3^{x-2}$

$\iff 3^{3x-3} = 3^x$

$\iff 3x-3 = x$

$\iff 2x = 3 \implies \underline{S = \left\{\dfrac{3}{2}\right\}}$

**k)** $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

$3^{4x+2} - 36 \cdot 3^{2x+1} + 243 = 0$

$y = 3^{2x+1}$

$y^2 - 36y + 243 = 0$

$(y-9)(y-27) = 0$

$$y = \begin{cases} 9 \\ 27 \end{cases} \iff 3^{2x+1} = \begin{cases} 9 & \iff 2x+1 = 2 & \iff x = 1/2 \\ 27 & \iff 2x+1 = 3 & \iff x = 1 \end{cases}$$

$$\implies \quad \underline{S = \left\{ \tfrac{1}{2} ; 1 \right\}}$$

l) $\quad 5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

$\quad 5^{4x-6} - 120 \cdot 5^{2x-3} - 625 = 0$

$y = 5^{2x-3}$

$\quad y^2 - 120y - 625 = 0$

$\quad (y+5)(y-125) = 0$

$$y = \begin{cases} -5 \\ 125 \end{cases} \iff 5^{2x-3} = \begin{cases} -5 & \text{ʓ} \\ 125 & \iff 2x-3 = 3 \implies \underline{S = \{3\}} \end{cases}$$

## Ex 4.2.2

a) $\to$ e) , h) i) , n) $\to$ r) , w) $\to$ z) $\qquad$ voir solutions

f) $\log_3 (\sqrt{3}) = 1/2 \quad \iff \quad 3^{1/2} = \sqrt{3}$

g) $\log_{243} \left( \tfrac{1}{243} \right) = -1 \quad \iff \quad 243^{-1} = \tfrac{1}{243}$

j) $\log_4 (\sqrt{2}) = \tfrac{1}{4} \quad \iff \quad 4^{1/4} = (2^2)^{1/4} = 2^{1/2} = \sqrt{2}$

k) $\log_{\frac{1}{8}} (64) = -2 \quad \iff \quad \left( \tfrac{1}{8} \right)^{-2} = 8^2 = 64$

l) $\log_5 (0{,}04) = -2 \quad \iff \quad 5^{-2} = \tfrac{1}{5^2} = \tfrac{1}{25} = \tfrac{4}{100} = 0{,}04$

m) $\log_3 (\sqrt[4]{27}) = 3/4 \quad \iff \quad 3^{3/4} = \sqrt[4]{3^3} = \sqrt[4]{27}$

s) $\log_2 \left( \tfrac{1}{8} \right) = -3 \quad \iff \quad 2^{-3} = \tfrac{1}{2^3} = \tfrac{1}{8}$

t) $\log_3 (\sqrt[4]{3}) = \frac{1}{4}$ $\iff$ $3^{1/4} = \sqrt[4]{3}$

u) $\log(200) - \log(2) = \log(100) = 2$

v) $\log_6(4) + \log_6(9) = \log_6(36) = 2$