EX 2.1.3

a) $d: \quad y = 2x - 3 \quad \Leftrightarrow \quad 2x - y - 3 = 0$

$\gamma: \quad x^2 + y^2 - 3x + 2y = 3$

$\Leftrightarrow \quad x^2 - 3x + \dfrac{9}{4} + y^2 + 2y + 1 = 3 + \dfrac{9}{4} + 1$

$\Leftrightarrow \quad \left(x - \dfrac{3}{2}\right)^2 + (y+1)^2 = \dfrac{25}{4}$

$\Rightarrow \quad C\left(\dfrac{3}{2}; -1\right) \quad \text{et} \quad r = \dfrac{5}{2}$

$\delta(C; d) = \dfrac{\left| 2 \cdot \dfrac{3}{2} - (-1) - 3 \right|}{\sqrt{2^2 + (-1)^2}} = \dfrac{|3+1-3|}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

$\cong 0{,}45 < \dfrac{5}{2}$

$\Rightarrow \quad \text{sécants}$

b)

$$x^2 - 8x + \textcolor{red}{16} + y^2 + 2y + \textcolor{red}{1} = -12 + \textcolor{red}{16} + \textcolor{red}{1}$$

$$(x-4)^2 + (y+1)^2 = 5$$

$$C(4,-1) \quad \text{et} \quad r = \sqrt{5}$$

$$d: \quad x - 2y - 1 = 0$$

$$\delta(C; d) = \frac{|4 - 2\cdot(-1) - 1|}{\sqrt{1+4}} = \frac{5}{\sqrt{5}} = \sqrt{5} = r$$

$$\Rightarrow \quad \text{tangents}$$

c) $\quad x^2 + y^2 = 1 \quad \Rightarrow \quad C(0,0) \quad \text{et} \quad r = 1$

$$d: \quad x - y + 10 = 0$$

$$\Rightarrow \quad \delta(C; d) = \frac{|10|}{\sqrt{1+1}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \cong 7,07 > 1$$

$$\Rightarrow \quad \text{disjoints}$$

Pour trouver les points d'intersection :

a) $\begin{cases} y = 2x - 3 \\ x^2 + y^2 - 3x + 2y = 3 \end{cases}$) par substitution

$x^2 + (2x-3)^2 - 3x + 2(2x-3) = 3$

$x^2 + 4x^2 - 12x + 9 - 3x + 4x - 6 - 3 = 0$

$5x^2 - 11x = 0$

$x(5x - 11) = 0$

$\Rightarrow \begin{cases} x = 0 \qquad \Rightarrow y = 2 \cdot 0 - 3 = -3 \qquad \Rightarrow I(0; -3) \\ x = \dfrac{11}{5} \qquad \Rightarrow y = 2 \cdot \dfrac{11}{5} - 3 = \dfrac{7}{5} \qquad \Rightarrow J\left(\dfrac{11}{5}; \dfrac{7}{5}\right) \end{cases}$

b) $\begin{cases} x - 2y - 1 = 0 \\ x^2 + y^2 - 8x + 2y + 12 = 0 \end{cases}$ $\iff$ $x = 2y + 1$

$\implies (2y+1)^2 + y^2 - 8(2y+1) + 2y + 12 = 0$

$4y^2 + 4y + 1 + y^2 - 16y - 8 + 2y + 12 = 0$

$5y^2 - 10y + 5 = 0$

$y^2 - 2y + 1 = 0$ $\left(\text{ou} \quad \Delta = \ldots \right)$

$(y-1)^2 = 0$

$y = 1 \implies X = 2 \cdot 1 + 1 = 3 \implies T(3;1)$