Ex 4.2.4

a) $\log(16) + 2\log(3) - 2\log(2) - \frac{1}{2}\log(9)$

$= \log(16) + \log(3^2) - \log(2^2) - \log(9^{1/2})$

$= \log(16) + \log(9) - \log(4) - \log(3)$

$= \log(16\cdot 9) - (\log(4) + \log(3))$

$= \log(16\cdot 9) - \log(4\cdot 3) = \log\left(\frac{16\cdot 9}{4\cdot 3}\right) = \underline{\log(12)}$

b) $\log(15) + 3\log(10) - \log(30) - \log(5)$

$= \log(15) + \log(10^3) - (\log(30) + \log(5))$

$= \log(15\cdot 10^3) - \log(30\cdot 5) = \log\left(\frac{15\cdot 10^3}{30\cdot 5}\right) = \log(100) = \underline{2}$

c) $4\log(5) + \log\left(\frac{1}{5}\right) - 3\log(3) + \frac{1}{3}\log(27)$

$= \log(5^4) - \log(5) - \log(3^3) + \log(\underbrace{27^{1/3}}_{3})$

$= \log\left(\frac{5^4\cdot 3}{5\cdot 3^3}\right) = \underline{\log\left(\frac{125}{9}\right)}$

d) $\dfrac{\log(20) + \log(100) - \log(2)}{\log(5'000) - \log(5) + \log(0{,}1)} = \dfrac{\log\left(\frac{20\cdot 100}{2}\right)}{\log\left(\frac{5000\cdot 0{,}1}{5}\right)} = \dfrac{\log(1000)}{\log(100)} = \underline{\frac{3}{2}}$

Ex 4.2.5

a) $x = \log_2(32) \Rightarrow S = \{5\}$

b) $2^x = 100 \Rightarrow S = \{\log_2(100)\}$

c) $\log_x(256) = 4$

$x^4 = 256$

$x = \sqrt[4]{256} = 4 \Rightarrow S = \{4\}$

d) $\log_2(x) = 4$

$2^4 = x \Rightarrow S = \{16\}$

e) $10^x = 5 \Rightarrow S = \{\log(5)\}$

f) $e^{2x-1} = 27$

$\Leftrightarrow 2x-1 = \ln(27)$

$\Leftrightarrow 2x = \ln(27)+1$

$\Leftrightarrow x = \frac{\ln(27)+1}{2} \Rightarrow S = \left\{\frac{\ln(27)+1}{2}\right\}$

g) $\log_x(1000) = 3$

$x^3 = 1000$

$x = 10 \Rightarrow S = \{10\}$

h) $12^x = -49$ ϟ $\Rightarrow S = \emptyset$

Ex 4.2.6

a) $\log_{11}(x+1) = \log_{11}(7) \Leftrightarrow x+1 = 7 \Leftrightarrow x = 6$

vérif : $\log_{11}(7) = \log_{11}(7)$ ✓ $\Rightarrow S = \{6\}$

b) $\log_6(2x-3) = \log_6(12) - \log_6(3)$

$\Leftrightarrow \log_6(2x-3) = \log_6\left(\frac{12}{3}\right)$

$\Leftrightarrow 2x-3 = 4$

$\Leftrightarrow x = \frac{7}{2}$

vérif : $\log_6(7-3) = \log_6(12) - \log_6(3)$ ✓ $\Rightarrow S = \left\{\frac{7}{2}\right\}$

c) $\log(x) - \log(x+1) = 3\log(4)$

$\Leftrightarrow \log\left(\frac{x}{x+1}\right) = \log(4^3)$

$\Leftrightarrow \frac{x}{x+1} = 64 \quad \Big| \cdot (x+1)$

$\Leftrightarrow x = 64(x+1)$

$\Leftrightarrow x = 64x + 64$

$\Leftrightarrow -63x = 64$

$\Leftrightarrow x = -\frac{64}{63}$ vérif : $\log\left(-\frac{64}{63}\right)$ ↯ $\Rightarrow S = \emptyset$

d) $2\log_3(x) = 3\log_3(5)$

$\Leftrightarrow \log_3(x^2) = \log_3(5^3)$

$\Leftrightarrow x^2 = 125$

$\Leftrightarrow x = \pm\sqrt{125} = \pm 5\sqrt{5}$ vérif : $2\log_3(\sqrt{125}) = 3\log_3(5)$ ✓

$2\log_3(-\sqrt{125})$ ↯

$\Rightarrow S = \{5\sqrt{5}\}$

e) $\ln(x) + \ln(x-2) = 0{,}5\ln(9)$

$\Leftrightarrow \ln\left(x(x-2)\right) = \ln\left(9^{0{,}5}\right) \qquad \left(9^{1/2} = 3\right)$

$\Leftrightarrow x(x-2) = 3$

$\Leftrightarrow x^2 - 2x = 3$

$\Leftrightarrow x^2 - 2x - 3 = 0$

$\Leftrightarrow (x-3)(x+1) = 0$

$\Leftrightarrow x = 3$ ou $x = -1$

vérif : $\ln(3) + \ln(1) = 0{,}5\ln(9)$ ✓

$\ln(-1)$ ϟ

$\Rightarrow S = \{3\}$

f) $\log_8(x+4) = 1 - \log_8(x-3)$

$\Leftrightarrow \log_8(x+4) + \log_8(x-3) = 1$

$\Leftrightarrow \log_8\left((x+4)(x-3)\right) = 1$

$\Leftrightarrow (x+4)(x-3) = 8$

$\Leftrightarrow x^2 + x - 12 = 8$

$\Leftrightarrow x^2 + x - 20 = 0$

$\Leftrightarrow (x-4)(x+5) = 0$

$\Leftrightarrow x = 4$ ou $x = -5$

vérif : $\log_8(8) = 1 - \log_8(1)$ ✓

$\log_8(-1)$ ϟ

$\Rightarrow S = \{4\}$

Variante :

$\Leftrightarrow \log_8(x+4) = \log_8(8) - \log_8(x-3)$

$\Leftrightarrow \log_8(x+4) = \log_8\left(\frac{8}{x-3}\right)$

$\Leftrightarrow x+4 = \frac{8}{x-3}$