Ex 2.3.1

a) $x^3 - 8x^2 + 16x - 5$ | $x-5$

$-x^3 + 5x^2$ | $x^2 - 3x + 1$

$-3x^2 + 16x - 5$

$+3x^2 \mp 15x$

$x - 5$

$x - 5$

$/$

$\Rightarrow x^3 - 8x^2 + 16x - 5 = (x-5)(x^2-3x+1)$

b) $x^5 - x^4 - 4x^3 + 2x^2 + 3x - 1$ | $x^2 + 2x - 1$

$-x^5 \mp 2x^4 + x^3$ | $x^3 - 3x^2 + 3x - 7$

$-3x^4 - 3x^3 + 2x^2 + 3x - 1$

$+3x^4 + 6x^3 \mp 3x^2$

$3x^3 - x^2 + 3x - 1$

$-3x^3 \mp 6x^2 + 3x$

$-7x^2 + 6x - 1$

$+7x^2 + 14x \mp 7$

$20x - 8$

$\Rightarrow x^5 - x^4 - 4x^3 + 2x^2 + 3x - 1 = (x^2+2x-1)(x^3-3x^2+3x-7) + 20x - 8$

c) $x^4 - 3x^3 + x - 5$ | $x^2 - 3$

$-x^4 + 3x^2$ | $x^2 - 3x + 3$

$-3x^3 + 3x^2 + x - 5$

$+3x^3 \mp 9x$

$3x^2 - 8x - 5$

$-3x^2 + 9$

$-8x + 4$

$\Rightarrow x^4 - 3x^3 + x - 5 = (x^2-3)(x^2-3x+3) - 8x + 4$

d)

$$\begin{array}{r|l} 35x^3+47x^2+13x+1 & 5x+1 \\ \hline -35x^3 \mp 7x^2 & 7x^2+8x+1 \\ \hline 40x^2+13x+1 & \\ -40x^2 \mp 8x & \\ \hline 5x+1 & \\ -5x \mp 1 & \\ \hline 0 & \end{array}$$

$$\Rightarrow 35x^3+47x^2+13x+1 = (5x+1)(7x^2+8x+1)$$

h)

$$\begin{array}{r|l} x^5-3x^3+2x^2+5x & x+2 \\ \hline -x^5 \mp 2x^4 & x^4-2x^3+x^2+5 \\ \hline -2x^4-3x^3+2x^2+5x & \\ +2x^2+4x^3 & \\ \hline x^3+2x^2+5x & \\ -x^3 \mp 2x^2 & \\ \hline 5x & \\ -5x \mp 10 & \\ \hline -10 & \end{array}$$

$$\Rightarrow x^5-3x^3+2x^2+5x = (x+2)(x^4-2x^3+x^2+5)-10$$

avec schéma de Horner :

	1	0	-3	2	5	0
-2		-2	4	-2	0	-10
	1	-2	1	0	5	-10

Ex 2.3.2

b)

$$\begin{array}{rl|l}
12x^4+47x^3+10x^2 & +12 & -3x^2-8x+6 \\
-12x^4\mp 32x^3+24x^2 & & \hline -4x^2-5x+2 \\
\hline
15x^3+34x^2 & +12 & \\
-15x^3\mp 40x^2+30x & & \\
\hline
-6x^2+30x & +12 & \\
+6x^2+16x\mp 12 & & \\
\hline
46x & &
\end{array}$$

$$12x^4+47x^3+10x^2+12 = (-3x^2-8x+6)(-4x^2-5x+2)+46x$$

Ex 2.3.3

$$(x-5)\cdot p = x^3-3x^2-4x-30$$

$$p = (x^3-3x^2-4x-30)\div(x-5)$$

Avec le schéma de Horner :

	1	-3	-4	-30
5		5	10	30
	1	2	6	0

$$\Rightarrow \quad p = x^2+2x+6$$

Ex 2.3.4

$$\begin{aligned}
p &= (2x^2+1)(5x^2-3x+1)+1-x \\
&= 10x^4-6x^3+2x^2+5x^2-3x+1+1-x \\
&= 10x^4-6x^3+7x^2-4x+2
\end{aligned}$$

Ex 2.3.8

avec Horner

a)

	4	-10	11	-5
1		4	-6	5
	4	-6	5	0

ou avec théorème du reste

$r = a(1) = 4 - 10 + 11 - 5 = 0$

b)

	9	1	-1	1	2
-2		-18	34	-66	130
	9	-17	33	-65	132

ou $r = a(-2) = 9 \cdot 16 - 8 - 4 - 2 + 2 = 132$

c)

	4	-5	3	-7
0		0	0	0
	4	-5	3	-7

ou $r = a(0) = -7$

Ex 2.3.9

a) $r = A(3) = 2 \cdot 27 - 9 + 15 - 1 = 59$

b) $r = A(-2) = 16 + 2 + 1 = 19$

c) $r = A(3) = 27 - 27 = 0$

Ex 2.3.12

a) $P(x) = 2x^3 - 14x + 12$ candidats : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$P(1) = 2 - 14 + 12 = 0 \Rightarrow$ 1 est un zéro

$P(2) = 16 - 28 + 12 = 0 \Rightarrow$ 2 est un zéro

$P(3) = 54 - 42 + 12 \neq 0$

$P(-3) = -54 + 42 + 12 = 0 \Rightarrow$ -3 est un zéro

b) $P(x) = x^4 - 6x^3 + x - 6$ candidats : $\pm 1, \pm 2, \pm 3, \pm 6$

$P(1) = 1 - 6 + 1 - 6 \neq 0$

$P(-1) = 1 + 6 - 1 - 6 = 0 \Rightarrow$ -1 est un zéro

$P(2) = 16 - 48 + 2 - 6 \neq 0$

$P(-2) = 16 + 48 - 2 - 6 \neq 0$

$P(3) = 81 - 162 + 3 - 6 \neq 0$

$P(-3) \neq 0$

$P(6) = 1296 - 1296 + 6 - 6 = 0 \Rightarrow$ 6 est un zéro

$P(-6) \neq 0$

Variante : $x^4 - 6x^3 + x - 6 \overset{GR}{=} x^3(x-6) + (x-6) = (x-6)(x^3+1)$

$= (x-6)(x+1)(x^2-x+1)$

6 ; -1 ; $\Delta < 0$

Ex 2.3.16

$p(x) = 2x^3 - 8x^2 + 8x \overset{MEE}{=} 2x(x^2 - 4x + 4) \overset{PR}{=} 2x(x-2)^2$

Ex 2.3.18

a) $p(x) = x^4 + 2x^3 - 5x^2 - 6x$

$\overset{\text{HEE}}{=} x(x^3 + 2x^2 - 5x - 6)$ candidats : $\pm 1, \pm 2, \pm 3, \pm 6$

$p(1) = 1 + 2 - 5 - 6 \neq 0$

$p(-1) = -1 + 2 + 5 - 6 = 0$ ✓

	1	2	−5	−6
−1		−1	−1	6
	1	1	−6	0

$\overset{\text{DIV}}{\Rightarrow} p(x) = x(x+1)(x^2 + x - 6) \overset{\text{SP}}{=} x(x+1)(x+3)(x-2)$

b) $x^5 + 3x^4 - 16x - 48$

$\overset{\text{GR}}{=} x^4(x+3) - 16(x+3)$

$= (x+3)(x^4 - 16)$

$\overset{\text{PR}}{=} (x+3)(x^2+4)(x^2-4)$

$\overset{\text{PR}}{=} (x+3)(x^2+4)(x+2)(x-2)$

c) $6x^4 - 5x^3 - 23x^2 + 20x - 4$ candidats : $\pm 1, \pm 2, \pm 4$

$p(1) = 6 - 5 - 23 + 20 - 4 \neq 0$

$p(-1) = 6 + 5 - 23 - 20 - 4 \neq 0$

$p(2) = 6 \cdot 16 - 5 \cdot 8 - 23 \cdot 4 + 20 \cdot 2 - 4 = 0$ ✓

	6	−5	−23	20	−4
2		12	14	−18	4
	6	7	−9	2	0
−2		−12	10	−2	
	6	−5	1	0	

$\Rightarrow p(x) = (x-2)(\underbrace{6x^3 + 7x^2 - 9x + 2}_{q(x)})$

candidats : $\underbrace{\pm 1}_{\text{Non car déjà essayé}}, \pm 2$

$q(2) = 6 \cdot 8 + 7 \cdot 4 - 9 \cdot 2 + 2 \neq 0$

$q(-2) = -6\cdot 8 + 7\cdot 4 + 9\cdot 2 + 2 = 0$

$\Rightarrow \quad p(x) = (x-2)(x+2)(6x^2-5x+1)$

$\Delta = 25 - 4\cdot 6 = 1$

$x_{1,2} = \dfrac{5 \pm 1}{12} = \begin{cases} 1/2 \\ 1/3 \end{cases}$

$\Rightarrow 6\left(x-\frac{1}{2}\right)\left(x-\frac{1}{3}\right) = (2x-1)(3x-1)$

$\Rightarrow \quad p(x) = (x-2)(x+2)(2x-1)(3x-1)$

Ex 2.3.19

a)

	1	1	1	1	1
1		1	2	3	4
	1	2	3	4	5

$\Rightarrow \quad q(x) = x^3 + 2x^2 + 3x + 4$ et $r = 5$

b)

	1	0	0	0	0	1
-1		-1	1	-1	1	-1
	1	-1	1	-1	1	0

$\Rightarrow \quad q(x) = x^4 - x^3 + x^2 - x + 1$ et $r = 0$

c)

	3	-8	7	1	-5	6
-2		-6	28	-70	138	-266
	3	-14	35	-69	133	-260

$\Rightarrow \quad q(x) = 3x^4 - 14x^3 + 35x^2 - 69x + 133$ et $r = -260$

Ex 2.3.20

$p(x) = x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

$r = p(1) = 1 - 6 + 15 - 20 + 15 - 6 + 1 = 0$ ✓ $\Rightarrow$ divisible par $x-1$

#

Ex 2.3.21

d)

$$\begin{array}{r} 6x^4 + 4x^3 - 7x^2 \\ -6x^4 \qquad\quad +9x^2 \\ \hline 4x^3 + 2x^2 \\ -4x^3 \qquad +6x \\ \hline 2x^2 + 6x \\ -2x^2 \qquad +3 \\ \hline 6x+3 \end{array} \qquad \begin{array}{|l} 2x^2-3 \\ \hline 3x^2+2x+1 \end{array}$$

$\Rightarrow \quad 6x^4 + 4x^3 - 7x^2 = (2x^2-3)(3x^2+2x+1) + 6x+3$

e)

$$\begin{array}{r} 14x^4 - 27x^3 + 21x^2 - 3x - 2 \\ -14x^4 + 21x^3 \mp 14x^2 \\ \hline -6x^3 + 7x^2 - 3x \\ +6x^3 \mp 9x^2 + 6x \\ \hline -2x^2 + 3x - 2 \\ -2x^2 + 3x - 2 \\ \hline 0 \end{array} \qquad \begin{array}{|l} 2x^2-3x+2 \\ \hline 7x^2-3x-1 \end{array}$$

$\Rightarrow \quad 14x^4 - 27x^3 + 21x^2 - 3x - 2 = (2x^2-3x+2)(7x^2-3x-1)$

Ex 2.3.22

$p(x) = 2x^3 + 3x^2 - 11x - 6$

candidats : $\pm 1, \pm 2, \pm 3, \pm 6$

$p(-1) \neq 0$, $p(-2) \neq 0$, $p(-3) = 0$ ✓

	2	3	−11	−6
−3		−6	9	6
	2	−3	−2	0

$\Rightarrow p(x) = (x+3)(2x^2-3x-2) = (x+3)2(x+\frac{1}{2})(x-2) = \underline{(x+3)(2x+1)(x-2)}$

$\Delta = 25$

$x_{1,2} = \frac{3 \pm 5}{4} = \begin{cases} -\frac{1}{2} \\ 2 \end{cases}$

Ex 2.3.23

a) $p(x) = x^3 + 9x^2 + 11x - 21$

candidats : $\pm 1, \pm 3, \pm 7, \pm 21$

$p(1) = 1 + 9 + 11 - 21 = 0$ ✓

	1	9	11	−21
1		1	10	21
	1	10	21	0

$p(x) \overset{\text{DIV}}{=} (x-1)(x^2+10x+21)$

$\overset{\text{SP}}{=} \underline{(x-1)(x+3)(x+7)}$

b) $p(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$

candidats : $\pm 1, \pm 3, \pm 5, \pm 15$

$p(1) = 1 + 2 - 16 - 2 + 15 = 0$ ✓

	1	2	−16	−2	15
1		1	3	−13	−15
	1	3	−13	−15	0
−1		−1	−2	15	
	1	2	−15	0	

$\Rightarrow p(x) \overset{\text{DIV}}{=} (x-1)(\underbrace{x^3+3x^2-13x-15}_{q(x)})$

candidats : $\pm 1, \pm 3, \pm 5, \pm 15$

$q(1) = 1 + 3 - 13 - 15 \neq 0$

$q(-1) = -1 + 3 + 13 - 15 = 0$ ✓

$\Rightarrow p(x) \overset{\text{DIV}}{=} (x-1)(x+1)(x^2+2x-15)$

$\overset{\text{SP}}{=} \underline{(x-1)(x+1)(x+5)(x-3)}$

c) idem ex 2.3.18 c)

d) $p(x) = x^5 - 3x^4 - 21x^3 + 43x^2 + 96x - 180$

candidats : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 9, \pm 10, \pm 12, \pm 15,$

$\pm 18, \pm 20, \pm 30, \pm 36, \pm 45, \pm 60, \pm 90, \pm 180$

$p(1) = 1 - 3 - 21 + 43 + 96 - 180 \neq 0$
$p(-1) = -1 - 3 + 21 + 43 - 96 - 180 \neq 0$
$p(2) = \ldots = 0$ ✓

	1	-3	-21	43	96	-180
2		2	-2	-46	-6	180
	1	-1	-23	-3	90	0
-3		-3	12	33	-90	
	1	-4	-11	30	0	
5		5	5	-30		
	1	1	-6	0		

$q(x) = x^4 - x^3 - 23x^2 - 3x + 90$

$q(-2) = \ldots \neq 0$
$q(3) = \ldots \neq 0$
$q(-3) = \ldots = 0$ ✓

$s(x) = x^3 - 4x^2 - 11x + 30$

$s(4) \ldots \neq 0$
$s(-4) \ldots \neq 0$
$s(5) = \ldots = 0$ ✓

$\Rightarrow p(x) \overset{\text{Div}}{=} (x-2)(x+3)(x-5)(x^2+x-6)$

$\overset{\text{SP}}{=} (x-2)(x+3)(x-5)(x-2)(x+3) = \underline{(x-2)^2(x+3)^2(x-5)}$