Ex 2.3.15
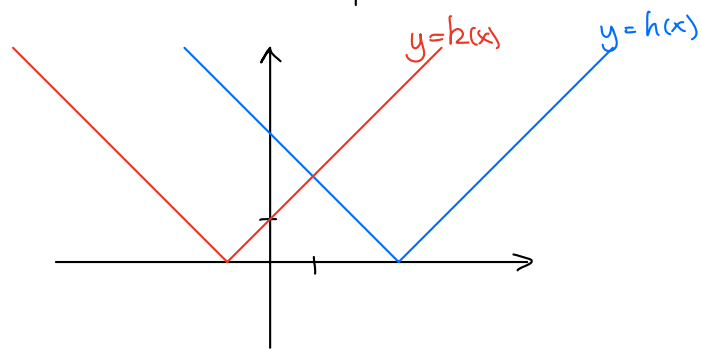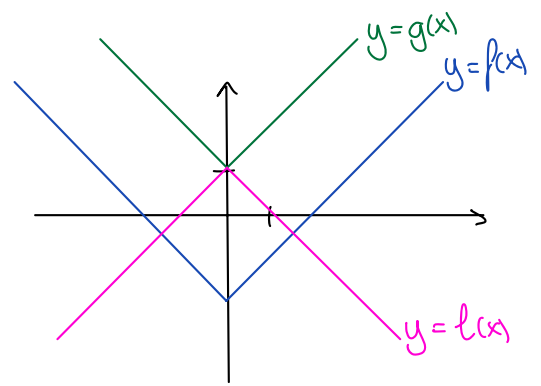
a) $f(x) = |x| - 2 = \begin{cases} -x-2 & \text{si } x < 0 \\ x-2 & \text{si } x \geqslant 0 \end{cases}$

$g(x) = |x| + 1 = \begin{cases} -x+1 & \text{si } x < 0 \\ x+1 & \text{si } x \geqslant 0 \end{cases}$

$h(x) = |x-3| = \begin{cases} -x+3 & \text{si } x < 3 \\ x-3 & \text{si } x \geqslant 3 \end{cases}$

$k(x) = |x+1| = \begin{cases} -x-1 & \text{si } x < -1 \\ x+1 & \text{si } x \geqslant 1 \end{cases}$

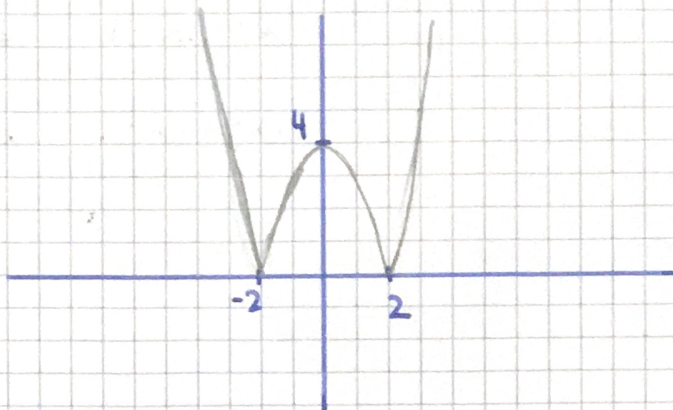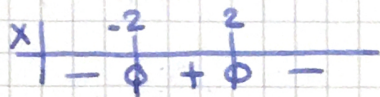$l(x) = -|x| + 1 = \begin{cases} -x+1 & \text{si } x < 0 \\ x+1 & \text{si } x \geqslant 0 \end{cases}$

EX 2.3.16

a) $f(x) = |4-x^2| = \begin{cases} 4-x^2 & \text{si } x \in [-2;2] \\ x^2-4 & \text{si } x \in]-\infty;-2[\,\cup\,]2;+\infty[\end{cases}$
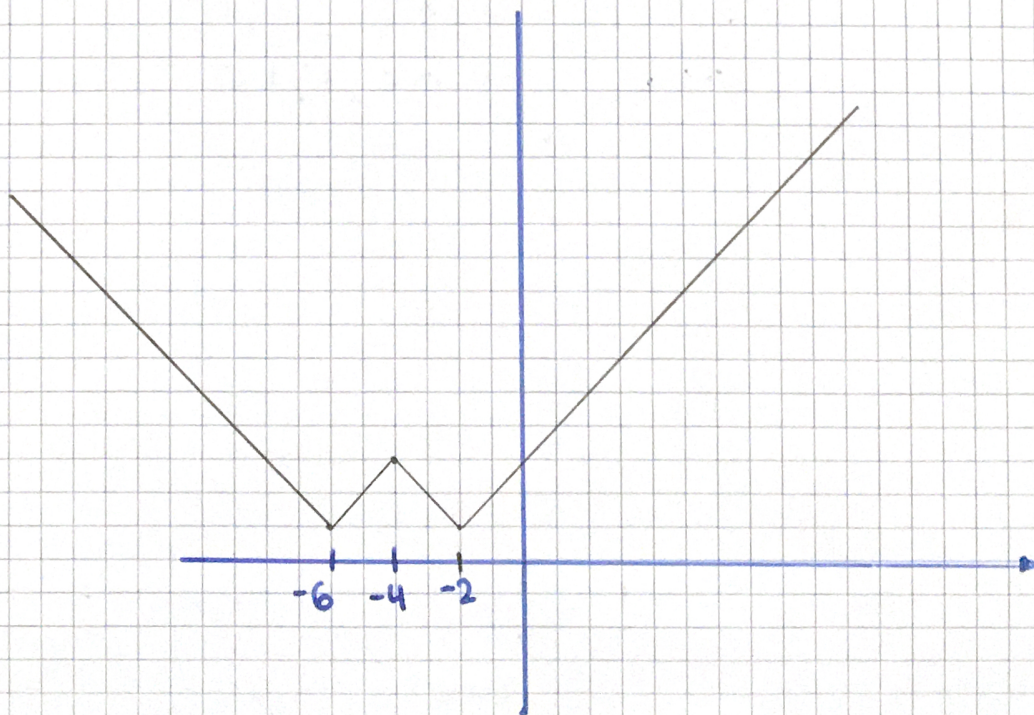
$4-x^2 = (2-x)(2+x)$

x		-2		2	
	$-$	0	$+$	0	$-$

b) $f(x) = \big|\,|x+4|-2\,\big|+1$

$= \begin{cases} |x+2|+1 & \text{si } x \geqslant -4 \\ \underbrace{|-x-6|}+1 & \text{si } x < -4 \\ \quad = |x-6| \end{cases} \qquad = $

$= \begin{cases} (x+2)+1 & \text{si } x \geqslant -2 \\ -(x+2)+1 & \text{si } -4 \leqslant x < -2 \\ (x+6)+1 & \text{si } -6 \leqslant x < -4 \\ -(x+6)+1 & \text{si } x < -6 \end{cases}$

$= \begin{cases} x+3 & \text{si } x \in [-2;+\infty[\\ -x-1 & \text{si } x \in [-4;-2[\\ x+7 & \text{si } x \in [-6;-4[\\ -x-5 & \text{si } x \in]-\infty;-6[\end{cases}$
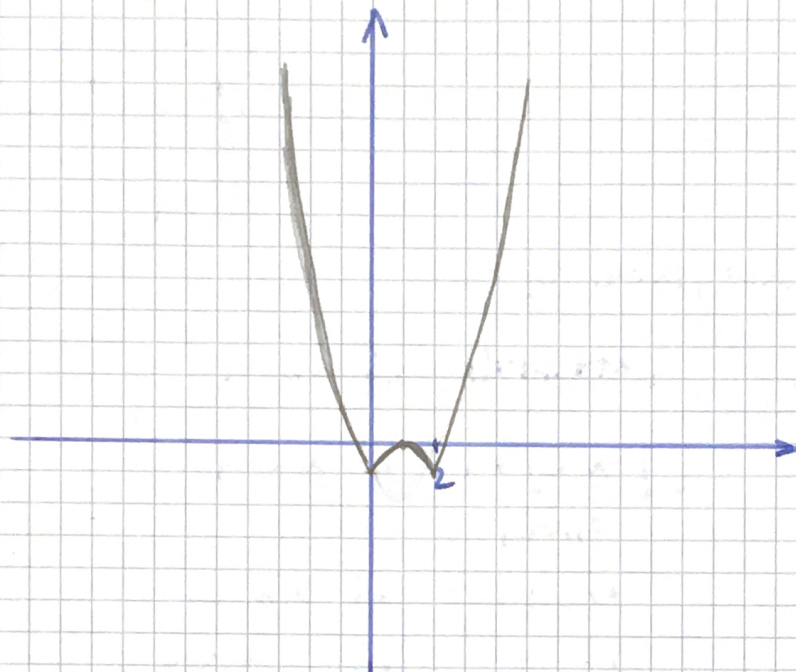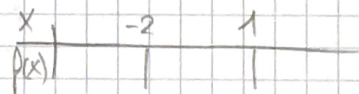
c) $f(x) = |x^2 - 2x| - 1$ $x^2 - 2x = x(x-2)$

x		0		2	
	$+$	0	$-$	0	$+$

$$= \begin{cases} x^2 - 2x - 1 & \text{si } x \in \,]-\infty; 0] \cup [2; +\infty[\\ -x^2 + 2x - 1 & \text{si } x \in \,]0; 2[\end{cases}$$

d) $f(x) = |x-1| + |x+2|$

x		-2		1	
$f(x)$					

$$= \begin{cases} -(x-1) - (x+2) & \text{si } x \in \,]-\infty; -2[\\ -(x-1) + (x+2) & \text{si } x \in [-2; 1[\\ (x-1) + (x+2) & \text{si } x \in [1; +\infty[\end{cases}$$

$$= \begin{cases} -2x - 1 & \text{si } x \in \,]-\infty; -2[\\ 3 & \text{si } x \in [-2; 1[\\ 2x + 1 & \text{si } x \in [1; +\infty \end{cases}$$