Ex 1

a) $f(x) = \sqrt{x^2-5x+4}$

cond : $x^2-5x+4 \geq 0 \Leftrightarrow (x-4)(x-1) \geq 0$

x		1		4	
$sgn(x^2-5x+4)$	$+$	0	$-$	0	$+$

* $ED(f) =]-\infty; 1] \cup [4; +\infty[$
*

x		1		4	
$sgn(f)$	$+$	0	///////	0	$+$

b) $f(x) = \sqrt{\dfrac{x^2-1}{x+2}}$

cond : $\dfrac{x^2-1}{x+2} \geq 0 \Leftrightarrow \dfrac{(x+1)(x-1)}{x+2} \geq 0$

x		-2		-1		1	
$sgn\left(\frac{x^2-1}{x+2}\right)$	$-$	$\parallel$	$+$	0	$-$	0	$+$

en 1000 $\dfrac{+}{+}$

* $ED(f) =]-2;-1] \cup [1;+\infty[$
*

x		-2		-1		1	
$sgn(f)$	///	$\parallel$	$+$	0	//////	0	$+$

c) $f(x) = \dfrac{\sqrt{-x^2-6x-5}}{x+3}$

cond : $-x^2-6x-5 \geq 0$ et $x+3 \neq 0$

$-(x+5)(x+1) \geq 0$ et $x \neq -3$

x		-5		-1	
$sgn(-x^2-6x-5)$	$-$	0	$+$	0	$-$

* $ED(f) = [-5;-1] - \{-3\} = [-5;-3[\cup]-3;-1[$
*

x		-5		-3		-1	
$\sqrt{-x^2-6x-5}$	////	0	$+$	$+$	$+$	0	////
$x+3$	////		$-$	0	$+$		////
$sgn(f)$	////	0	$-$	$\parallel$	$+$	0	////

du même signe que $x-3$ car $\sqrt{-x^2-6x-5}$ est tjrs positif.

Ex2

a) $f(x) = \dfrac{\log(1-x^2)}{x^2}$

cond : $1-x^2 > 0$ et $x^2 \neq 0$

$(1+x)(1-x) > 0$ et $x \neq 0$

x		-1		1	
$\text{sgn}(1-x^2)$	$-$	0	$+$	0	$-$

$ED(f) =]-1;1[- \{0\} =]-1;0[\cup]0;+\infty[$

b) $f(x) = \dfrac{x^2+6}{2-\ln(x+4)}$

cond : $x+4 > 0$ et $2-\ln(x+4) \neq 0$

$x > -4$ et $2 \neq \ln(x+4)$

$e^2 \neq x+4$

$e^2-4 \neq x$ $(\cong 3,4)$

$ED(f) =]-4;+\infty[- \{e^2-4\} =]-4;e^2-4[\cup]e^2-4;+\infty[$

c) $f(x) = \tan(2x+3)$

cond : $2x+3 \neq \dfrac{\pi}{2} + k\cdot\pi$, $k \in \mathbb{Z}$

$2x \neq \dfrac{\pi}{2} - 3 + k\pi$, $k \in \mathbb{Z}$

$x \neq \dfrac{\pi}{4} - \dfrac{3}{2} + \dfrac{k\pi}{2}$, $k \in \mathbb{Z}$

$ED(f) = \mathbb{R} - \left\{ \dfrac{\pi}{4} - \dfrac{3}{2} + k\dfrac{\pi}{2} \mid k \in \mathbb{Z} \right\}$

Ex3

$f(x) = \dfrac{x}{x+2}$ $\quad ED(f) = \mathbb{R} - \{-2\}$

$g(x) = 2x-3$ $\quad ED(g) = \mathbb{R}$

$h(x) = \sqrt{x-2}$ $\quad ED(h) = [2;+\infty[$

a) $(f+h)(x) = \dfrac{x}{x+2} + \sqrt{x-2}$ $\quad ED(f+h) = [2;+\infty[$ $(= ED(f) \cap ED(h))$

b) $\left(\frac{g}{f}\right)(x) = \frac{2x-3}{\frac{x}{x+2}} = 2x-3 \cdot \frac{x+2}{x} = \frac{(2x-3)(x+2)}{x}$

$ED\left(\frac{g}{f}\right) = \mathbb{R}^* - \{-2\} = ED(g) \cap ED(f) - \underbrace{\{x \mid f(x)=0\}}_{\text{zéro de } f.}$

c) $(g \cdot h)(x) = (2x-3)\sqrt{x-2}$ $\qquad ED(g \cdot h) = [2\,;+\infty[\;\left(= ED(g) \cap ED(h)\right)$

d) $(g \circ g)(x) = g(g(x)) = g(2x-3) = 2(2x-3)-3 = 4x-9$

$ED(g \circ g) = \mathbb{R}$

e) $(h \circ g)(x) = h(g(x)) = h(\overbrace{2x-3}^{\in ED(h)}) = \sqrt{(2x-3)-2} = \sqrt{2x-5}$

$\Downarrow$ $\qquad\qquad\qquad\qquad \Downarrow$

$2x-3 \geq 2$ $\qquad\qquad 2x-5 \geq 0$

$x \geq 5/2$ $\qquad\qquad x \geq 5/2$

$ED(h \circ g) = \left[\frac{5}{2}\,;+\infty\right[$

f) $(f \circ g)(x) = f(g(x)) = f(\overbrace{2x-3}^{\in ED(f)}) = \frac{2x-3}{(2x-3)+2} = \frac{2x-3}{2x-1}$

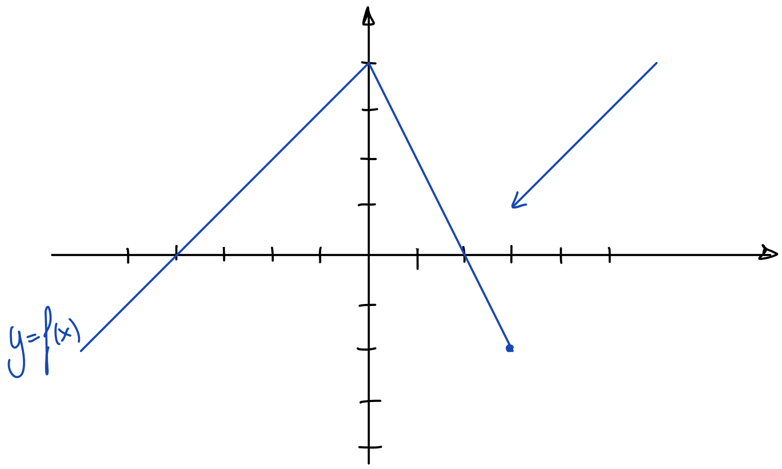
$\Downarrow$ $\qquad\qquad\qquad\qquad \Downarrow$

$2x-3 \neq -2$ $\qquad\qquad 2x-1 \neq 0$

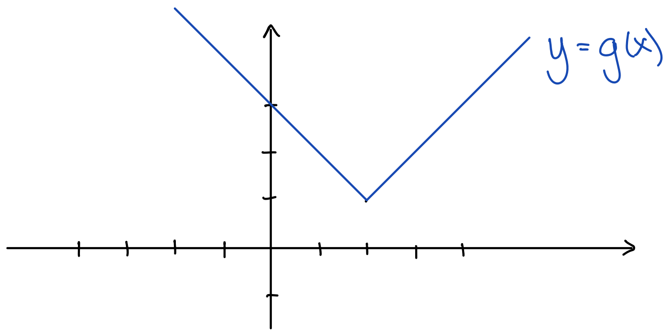
$x \neq 1/2$ $\qquad\qquad x \neq 1/2$

$ED(f \circ g) = \mathbb{R} - \left\{\frac{1}{2}\right\}$

Ex 4

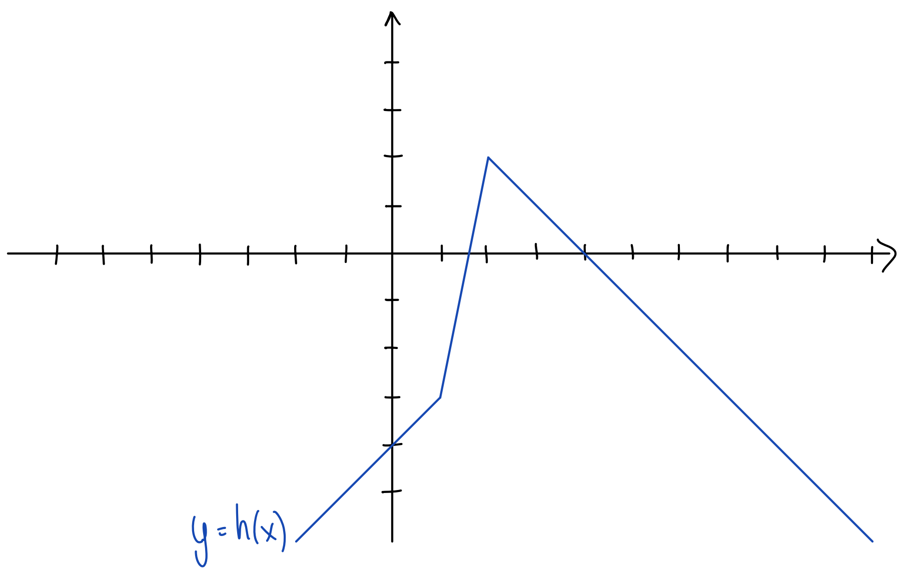


$$g(x) = |-x+2|+1 = \begin{cases} -(-x+2)+1 & \text{si } -x+2<0 \\ -x+2+1 & \text{si } -x+2 \geq 0 \end{cases} = \begin{cases} x-1 & \text{si } x>2 \\ -x+3 & \text{si } x \leq 2 \end{cases}$$



$$h(x) = 2|x-1| - 3|x-2|$$

$$= \begin{cases} -2(x-1)+3(x-2) & \text{si } x<1 \\ 2(x-1)+3(x-2) & \text{si } 1 \leq x < 2 \\ 2(x-1)-3(x-2) & \text{si } x \geq 2 \end{cases} = \begin{cases} x-4 & \text{si } x<1 \\ 5x-8 & \text{si } 1 \leq x < 2 \\ -x+4 & \text{si } x \geq 2 \end{cases}$$



Ex 5

$$f(x) = \begin{cases} -2x & \text{si } x \in]-\infty; 1] \\ -2 & \text{si } x \in]1; 5] \\ x-7 & \text{si } x \in]5; +\infty[\end{cases}$$

Ex 6

a) $f(x) = \dfrac{x^2+2}{x^2-1}$ $\quad ED(f) = \mathbb{R} - \{\pm 1\}$

$f(-x) = \dfrac{(-x)^2+2}{(-x)^2-1} = \dfrac{x^2+2}{x^2-1} = f(x) \Rightarrow$ f est paire

b) $f(x) = \sqrt{x-1}$ $\quad ED(f) = [1;+\infty[$

comme ED n'est pas sym. p.r. à 0, f est ni paire ni impaire

c) $f(x) = \dfrac{x}{x^3-1}$ $\quad ED(f) = \mathbb{R} - \{1\}$

comme ED n'est pas sym. p.r. à 0, f est ni paire ni impaire

d) $f(x) = |x^3+2x|$ $\quad ED(f) = \mathbb{R}$

$f(-x) = |(-x)^3 + 2(-x)| = |-x^3-2x| = |x^3+2x| = f(x) \Rightarrow$ f est paire

e) $f(x) = x\sin(x)$ $\quad ED(f) = \mathbb{R}$

$f(-x) = -x\sin(-x) = -x(-\sin(x)) = x\sin(x) = f(x) \Rightarrow$ f est paire

f) $f(x) = x^2\sin(x)$ $\quad ED(f) = \mathbb{R}$

$f(-x) = (-x)^2\sin(-x) = x^2(-\sin(x)) = -x^2\sin(x) = -f(x) \Rightarrow$ f est impaire

g) $f(x) = x + \sin(x)$ $\quad ED(f) = \mathbb{R}$

$f(-x) = -x + \sin(-x) = -x - \sin(x) = -(x+\sin(x)) = -f(x) \Rightarrow$ f est impaire.