Ex 1

a) $5^{x-3} = 625 \quad | \log_5()$

$\log_5(5^{x-3}) = \log_5(625)$

Prop. 1 $\quad x-3 = 4 \quad$ car $5^4 = 625$ ou à la m.à.c. $\frac{\log(625)}{\log(5)} = 4$

$x = 7$

$\Rightarrow \quad S = \{7\}$

Variante :

$5^{x-3} = 625$

$5^{x-3} = 5^4 \quad | \log_5()$

$x-3 = 4$

...

b) $6^{x+3} = 5 \quad | \log_6(\ )$

$\log_6(6^{x+3}) = \log_6(5)$

Prop. 1 $\quad x+3 = \log_6(5)$

$x = \log_6(5) - 3 \quad \Rightarrow \quad S = \{\log_6(5) - 3\}$

c) $e^{2x-1} = -8 \quad ⚡ \quad e^u > 0$

ou $\quad 2x-1 = \ln(-8) \quad ⚡ \quad \ln(u)$ défini si $u > 0 \quad \Rightarrow \quad S = \emptyset$

d) $e^{\frac{x}{10}} = 7 \quad | \ln()$

$\ln(e^{x/10}) = \ln(7)$

Prop. 1 $\quad \frac{x}{10} = \ln(7)$

$x = 10\ln(7) = \ln(7^{10}) \quad \Rightarrow \quad S = \{10\ln(7)\} = \{\ln(7^{10})\}$

e) $e^{(x^2)} + 1 = 8 \Leftrightarrow e^{(x^2)} = 7 \quad | \ln()$

$\ln(e^{(x^2)}) = \ln(7)$

Prop. 1 $\quad x^2 = \ln(7)$

$x = \pm\sqrt{\ln(7)} \quad \Rightarrow \quad S = \{-\sqrt{\ln(7)} ; \sqrt{\ln(7)}\}$

Ex 2

a) $\ln(e^4) = \underline{4}$

b) $\ln(\sqrt{e}) = \ln(e^{1/2}) = \underline{\frac{1}{2}}$

c) $\ln\left(\frac{1}{e^2}\right) = \ln(e^{-2}) = \underline{-2}$

d) $\ln(\underbrace{-e}_{<0})$ ϟ <u>impossible</u> car $\ln(u)$ défini seulement si $u>0$

e) $\ln(\underbrace{0}_{ϟ})$ <u>impossible</u> " " " "

f) $\ln(1) = \ln(e^0) = \underline{0}$

g) $\log_5\left(\frac{1}{125}\right) = \log_5(5^{-3}) = \underline{-3}$

h) $\log(0{,}01) = \log(10^{-2}) = \underline{-2}$

i) $\log(\sqrt[4]{1000}) = \log(\sqrt[4]{10^3}) = \log(10^{3/4}) = \underline{\frac{3}{4}}$

j) $\log_8(2) = \log_8(\sqrt[3]{8}) = \log_8(8^{1/3}) = \underline{\frac{1}{3}}$

k) $\log(4) + 2\log(5) \overset{\text{Prop 5.}}{=} \log(4) + \log(25) \overset{\text{Prop.3}}{=} \log(4\cdot 25) = \log(100)$
$= \log(10^2) = \underline{2}$

l) $\log(35) - \log(7) + \log(2) \overset{\text{Prop 3+4}}{=} \log\left(\frac{35}{7}\cdot 2\right) = \log(5\cdot 2) = \log(10) = \underline{1}$

Ex3

a) $6\log_7(x) + 1 = 11$

$$6\log_7(x) = 10$$

$$\log_7(x) = \frac{10}{6} = \frac{5}{3} \quad \Big| \ 7^{(\ )}$$

$$7^{\log_7(x)} = 7^{5/3}$$

Prop 2.

$$x = 7^{5/3}$$

vérif : $6 \cdot \log_7(7^{5/3}) + 1 = 6 \cdot \frac{5}{3} + 1 = 10+1 = 11 \checkmark$

$$\Rightarrow \underline{S = \left\{ 7^{5/3} \right\} = \left\{ \sqrt[3]{7^5} \right\} = \left\{ \sqrt[3]{16'807} \right\}}$$

b) $\log(x-4) = 4 \quad \Big| \ 10^{(\ )}$

$$10^{\log(x-4)} = 10^4$$

Prop 2.

$$x - 4 = 10'000$$

$$x = 10'004$$

vérif : $\log(10'000) = 4 \checkmark$ $\Rightarrow$ $\underline{S = \{10'004\}}$

c) $\ln(x^2) = 0 \quad \Big| \ e^{(\ )}$

Prop 2.

$$x^2 = e^0 = 1$$

$$x = \pm 1$$

vérif : $\ln(1^2) = \ln(1) = 0 \checkmark$

$\ln((-1)^2) =$ " " $\checkmark$

$\Rightarrow \underline{S = \{\pm 1\}}$

d) $\ln(2x) - \ln(5x-8) = 0 \quad \Big| \ e^{(\ )}$

$$\ln(2x) = \ln(5x-8)$$

Prop 2.

$$2x = 5x - 8$$

$$-3x = -8$$

$$x = \frac{8}{3}$$

vérif : $\ln\left(2 \cdot \frac{8}{3}\right) - \ln\left(5 \cdot \frac{8}{3} - 8\right) = \ln\left(\frac{16}{3}\right) - \ln\left(\frac{16}{3}\right) = 0 \checkmark$

$$\Rightarrow \underline{S = \left\{ \frac{8}{3} \right\}}$$

Variante : avec prop. 4 on obtient l'équation e)

e) $\ln\left(\frac{2x}{5x-8}\right) = 0$

Prop 2.

$$\frac{2x}{5x-8} = 1$$

$$2x = 5x-8$$

$$-3x = -8$$

$$x = \frac{8}{3}$$

vérif : $\ln\left(\frac{2 \cdot 8/3}{5 \cdot 8/3 - 8}\right) = \ln\left(\frac{16/3}{16/3}\right) = \ln(1) = 0 \checkmark$

$\Rightarrow$ $S = \left\{\frac{8}{3}\right\}$

f) $\frac{\ln(2x)}{\ln(5x-8)} = 0$

$$\ln(2x) = 0$$

$$2x = 1 \quad (= e^0)$$

$$x = \frac{1}{2}$$

vérif : $\frac{\ln(2 \cdot 1/2)}{\ln(5 \cdot 1/2 - 8)} = \frac{\ln(1)}{\ln(-\frac{11}{2})}$ $\Rightarrow$ $S = \emptyset$

↯ imp.

g) $\ln(x) + 1 = \ln(x+1)$

$$\ln(x) + \ln(e) = \ln(x+1)$$

$$\ln(ex) = \ln(x+1) \qquad | \; e^{(\,)}$$

$$ex = x+1$$

$$ex - x = 1$$

$$x(e-1) = 1$$

$$x = \frac{1}{e-1}$$

vérif : $\ln\left(\frac{1}{e-1}\right) + 1 = \ln(1) - \ln(e-1) + 1 = 1 - \ln(e-1)$

$\ln\left(\frac{1}{e-1} + 1\right) = \ln\left(\frac{1+e-1}{e-1}\right) = \ln\left(\frac{e}{e-1}\right)$ // $\checkmark$

$= \ln(e) - \ln(e-1) = 1 - \ln(e-1)$

$\Rightarrow$ $S = \left\{\frac{1}{e-1}\right\}$

Variante: $\ln(x) - \ln(x+1) = -1$

$$\ln\left(\frac{x}{x+1}\right) = -1$$

$$\frac{x}{x+1} = e^{-1} = \frac{1}{e} \qquad \Big| \cdot (x+1)$$

$$x = \frac{1}{e}(x+1)$$

$$x - \frac{1}{e}x = \frac{1}{e}$$

$$x\left(1-\frac{1}{e}\right) = \frac{1}{e}$$

$$x = \frac{1}{e} : \underbrace{\left(1-\frac{1}{e}\right)}_{\frac{e-1}{e}} = \frac{1}{e} \cdot \frac{e}{e-1} = \frac{1}{e-1}$$