Ex 1.1.1

a) $f(x) = e^{5x}$ $\qquad$ $\underline{ED(f) = \mathbb{R}}$

$\underline{f'(x) = 5e^{5x}}$

b) $f(x) = e^{x^2}$ $\qquad$ $\underline{ED(f) = \mathbb{R}}$

$\underline{f'(x) = 2x\, e^{x^2}}$

c) $f(x) = e^{1/x}$ $\qquad$ cond: $x \neq 0 \Rightarrow \underline{ED(f) = \mathbb{R}^*}$

$$\underline{f'(x) = -\frac{1}{x^2} e^{1/x} = -\frac{e^{1/x}}{x^2}}$$

d) $f(x) = e^{\sqrt{x^2+x}}$ $\qquad$ cond: $x^2+x \geq 0 \Leftrightarrow \underset{\downarrow\,0\quad\downarrow\,-1}{x(x+1)} \geq 0$

x		-1		0	
x^2+x	$+$	0	$-$	0	$+$

$\Rightarrow \underline{ED(f) = \left]-\infty; -1\right] \cup \left[0; +\infty\right[}$

$$\underline{f'(x) = \frac{2x+1}{2\sqrt{x^2+x}}\, e^{\sqrt{x^2+x}}}$$

(e) $f(x) = \exp\left(\sqrt{\frac{1+x^2}{1-x^2}}\right) = e^{\sqrt{\frac{1+x^2}{1-x^2}}}$

cond: $\frac{1+x^2}{1-x^2} \geq 0 \Leftrightarrow \frac{1+x^2}{(1+x)(1-x)} \geq 0$

x		-1		1	
$\frac{1+x^2}{1-x^2}$	$-$	$\Vert$	$+$	$\Vert$	$-$

$\Rightarrow \underline{ED(f) = \left]-1; 1\right[}$

$$f'(x) = e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{\frac{2x(1-x^2)+2x(1+x^2)}{(1-x^2)^2}}{2\sqrt{\frac{1+x^2}{1-x^2}}}$$

$$= e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{\cancel{2}x(1-x^2+1+x^2)}{(1-x^2)^2} \cdot \frac{1}{\cancel{2}\sqrt{\frac{1+x^2}{1-x^2}}}$$

$$\underline{= e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{2x}{(1-x^2)^2} \cdot \sqrt{\frac{1-x^2}{1+x^2}}}$$

(f) $f(x) = e^{\sin(x)}$ $\qquad ED(f) = \mathbb{R}$

$f'(x) = \cos(x)\, e^{\sin(x)}$

g) $f(x) = x^2 \cdot e^x$ $\qquad ED(f) = \mathbb{R}$ $\qquad u = x^2 \quad v = e^x$

$u' = 2x \quad v' = e^x$

$f'(x) = 2x \cdot e^x + x^2 \cdot e^x = xe^x(2+x)$

(h) $f(x) = e^{-x}\cos(x)$ $\qquad ED(f) = \mathbb{R}$ $\qquad u = e^{-x} \quad v = \cos(x)$

$u' = -e^{-x} \quad v' = -\sin(x)$

$f'(x) = -e^{-x}\cos(x) - e^{-x}\sin(x) = -e^{-x}(\cos(x) + \sin(x))$

Ex 1.1.6

a) $f(x) = \ln(5x)$ cond: $5x > 0 \Leftrightarrow x > 0 \Rightarrow ED(f) = \mathbb{R}^*_+$

$f'(x) = \frac{5}{5x} = \frac{1}{x}$

b) $f(x) = \ln(x-1)$ cond: $x-1 > 0 \Leftrightarrow x > 1 \Rightarrow ED(f) =]1; +\infty[$

$f'(x) = \frac{1}{x-1}$

c) $f(x) = \ln(1-x)$ cond: $1-x > 0 \Leftrightarrow 1 > x \Rightarrow ED(f) =]-\infty; 1[$

$f'(x) = \frac{-1}{1-x} = \frac{1}{x-1}$

d) $f(x) = \ln(|1-x|)$ cond: $1-x \neq 0 \Leftrightarrow x \neq 1 \Rightarrow ED(f) = \mathbb{R} - \{1\}$

$$f(x) = \begin{cases} \ln(x-1) & \text{si } 1-x<0 \\ \ln(1-x) & \text{si } 1-x>0 \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{1}{x-1} \\ \frac{-1}{1-x} \end{cases} \Rightarrow f'(x) = \frac{1}{x-1} \quad \forall x \in ED(f)$$

e) $f(x) = \ln(x^2-x)$ cond: $x^2-x > 0 \Leftrightarrow x(x-1) > 0$

x		0		1	
x^2-x	+	0	−	0	+

$\Rightarrow ED(f) =]-\infty; 0[\cup]1; +\infty[$

$f'(x) = \frac{2x-1}{x^2-x}$

f) $f(x) = \ln(x-x^2)$ cond: $x - x^2 > 0 \Leftrightarrow x(1-x) > 0$

x		0		1	
x^2-x	−	0	+	0	−

$\Rightarrow ED(f) =]0; 1[$

$f'(x) = \frac{1-2x}{x-x^2} = \frac{2x-1}{x^2-x}$

g) $f(x) = \ln(|x^2-x|)$ cond: $x^2-x \neq 0 \Leftrightarrow x(x-1) \neq 0 \Leftrightarrow x \neq 0, 1$

$\Rightarrow ED(f) = \mathbb{R}^* - \{1\}$

$f'(x) = \frac{2x-1}{x^2-x}$

h) $f(x) = \ln\left(\frac{x^2}{1-x}\right)$

cond. $\frac{x^2}{1-x} > 0$

x		0		1	
$\frac{x^2}{1-x}$	$+$	0	$+$	$\|\|$	$-$
		(2)			

$\Rightarrow ED(f) =]-\infty ; 1[- \{0\}$

$u = \frac{x^2}{1-x}$

$u' = \frac{2x(1-x)+1\cdot x^2}{(1-x)^2} = \frac{2x - 2x^2 + x^2}{(1-x)^2} = \frac{2x-x^2}{(1-x)^2} = \frac{x(2-x)}{(1-x)^2}$

$f'(x) = \frac{\frac{-x(x-2)}{(1-x)^2}}{\frac{x^2}{1-x}} = \frac{x(2-x)}{(1-x)^2} \cdot \frac{1-x}{x^2} = \frac{2-x}{x(1-x)} = \frac{x-2}{x(x-1)}$

i) $f(x) = \ln\left(\sqrt{3-x^2}\right)$

cond : $\underbrace{\sqrt{3-x^2} > 0 \quad \text{et} \quad 3-x^2 \geq 0}_{3-x^2>0}$

$\Leftrightarrow (\sqrt{3}+x)(\sqrt{3}-x) > 0$

x		$-\sqrt{3}$		$\sqrt{3}$	
$3-x^2$	$-$	0	$+$	0	$-$

$ED(f) =]-\sqrt{3} ; \sqrt{3}[$

$u = \sqrt{3-x^2} \qquad u' = \frac{-2x}{2\sqrt{3-x^2}} = \frac{-x}{\sqrt{3-x^2}}$

$f'(x) = \frac{\frac{-x}{\sqrt{3-x^2}}}{\sqrt{3-x^2}}$

$= \frac{-x}{\sqrt{3-x^2}} \cdot \frac{1}{\sqrt{3-x^2}} = \frac{-x}{3-x^2} = \frac{x}{x^2-3}$

j) $f(x) = \ln(3x^5) \qquad ED(f) = \mathbb{R}_+^*$

$f'(x) = \frac{15x^4}{3x^5} = \frac{5}{x}$

k) $f(x) = x\ln(x) - x \qquad ED(f) = \mathbb{R}_+^*$

$f'(x) = 1\cdot\ln(x) + x\cdot\frac{1}{x} - 1 = \ln(x) + 1 - 1 = \ln(x)$

l) $f(x) = \ln|\cos(x)|$

cond: $|\cos(x)| > 0 \Leftrightarrow \cos(x) \neq 0$

$ED(f) = \mathbb{R} - \left\{ \frac{\pi}{2} + k\cdot\pi \mid k \in \mathbb{Z} \right\}$

$$f(x) = \begin{cases} \ln(\cos(x)) & \text{si } \cos(x) > 0 \\ \ln(-\cos(x)) & \text{si } \cos(x) < 0 \end{cases}$$

$u = \cos(x)$, $u' = -\sin(x)$

$u = -\cos(x)$ $u' = \sin(x)$

$$f'(x) = \left\{ \begin{array}{lll} \frac{-\sin(x)}{\cos(x)} = -\tan(x) & \text{si } \cos(x) > 0 \\ \frac{\sin(x)}{-\cos(x)} = -\tan(x) & \text{si } \cos(x) < 0 \end{array} \right\} \Rightarrow f'(x) = -\tan(x)$$

m) $f(x) = \dfrac{x}{\ln(x)}$

cond: $\ln(x) \neq 0$ et $x > 0$

$x \neq 1$

$ED(f) = \mathbb{R}_+^* - \{1\}$

$u = x$ $\quad v = \ln(x)$

$u' = 1$ $\quad v' = \frac{1}{x}$

$$f'(x) = \frac{1\ln(x) - x\cdot\frac{1}{x}}{\ln^2(x)} = \frac{\ln(x) - 1}{\ln^2(x)}$$

n) $f(x) = \dfrac{1}{x\ln(x)}$

cond: $x\ln(x) \neq 0$ et $x > 0$

$\swarrow \quad \downarrow$

$0 \quad 1$

$ED(f) = \mathbb{R}_+^* - \{1\}$

$u = 1$ $\quad v = x\ln(x)$

$u' = 0$ $\quad v' = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x) + 1$

$$f'(x) = -\frac{\ln(x)+1}{x^2\ln^2(x)}$$