

Ex 2.3.25

$$a) \quad \underline{x^3 + 2x^2} - \underline{x - 2} = 0$$

$$\text{GR} \quad x^2(x+2) - (x+2) = 0$$

$$(x+2)(x^2-1) = 0$$

$$\text{PR} \quad (x+2)(x+1)(x-1) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ -2 & -1 & 1 \end{array}$$

$$\Rightarrow \underline{S = \{-2; -1; 1\}}$$

$$b) \quad \underline{x^3 - 3x^2} - \underline{4x + 12} = 0$$

$$\text{GR} \quad x^2(x-3) - 4(x-3) = 0$$

$$(x-3)(x^2-4) = 0$$

$$\text{PR} \quad (x-3)(x+2)(x-2) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 3 & -2 & 2 \end{array}$$

$$\Rightarrow \underline{S = \{-2; 2; 3\}}$$

$$c) \quad 4x^5 - 12x^4 + 9x^3 = 0$$

$$\text{HEE} \quad x^3(4x^2 - 12x + 9) = 0$$

$$\text{PR} \quad x^3(2x-3)^2 = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \\ 0 & 2x-3=0 & \\ & 2x=3 & \\ & x=3/2 & \end{array}$$

$$\Rightarrow \underline{S = \{0; \frac{3}{2}\}}$$

$$d) \quad 16x^3 - 16x^2 - 4x + 4 = 0$$

$$4(\underline{4x^3 - 4x^2} - \underline{x + 1}) = 0$$

$$4[4x^2(x-1) - (x-1)] = 0$$

$$4(x-1)(4x^2-1) = 0$$

$$4(x-1)(2x+1)(2x-1) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{array}$$

$$\Rightarrow \underline{S = \{-\frac{1}{2}; \frac{1}{2}; 1\}}$$

Ex 2.3.26

a) $x^4 + 2x^3 - 4x^2 - 5x - 6 = 0$

$$\begin{array}{r|rrrrr} & 1 & 2 & -4 & -5 & -6 \\ 2 & & 2 & 8 & 8 & 6 \\ \hline & 1 & 4 & 4 & 3 & 0 \end{array}$$

$\Rightarrow (x-2)(x^3 + 4x^2 + 4x + 3) = 0$

$$\begin{array}{r|rrrr} & 1 & 4 & 4 & 3 \\ -3 & & -3 & -3 & -3 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$\Rightarrow (x-2)(x+3)(x^2+x+1) = 0$
 $\downarrow \quad \downarrow \quad \Delta = -3 < 0$
 $2 \quad -3$

b) $x^4 - 7x^3 + 18x^2 - 20x + 8 = 0$

$$\begin{array}{r|rrrrr} & 1 & -7 & 18 & -20 & 8 \\ 1 & & 1 & -6 & 12 & -8 \\ \hline & 1 & -6 & 12 & -8 & 0 \end{array}$$

$\Rightarrow (x-1)(x^3 - 6x^2 + 12x - 8) = 0$

$(x-1)(x-2)^3 = 0$
 $\downarrow \quad \downarrow$
 $1 \quad 2$

c) $35x^3 + 47x^2 + 13x + 1 = 0$

$$\begin{array}{r|rrrr} & 35 & 47 & 13 & 1 \\ -1 & & -35 & -12 & -1 \\ \hline & 35 & 12 & 1 & 0 \end{array}$$

$\Rightarrow (x+1)(35x^2 + 12x + 1) = 0$

candidates: $\pm 1, \pm 2, \pm 3, \pm 6$

$p(1) = 1 + 2 - 4 - 5 - 6 \neq 0$
 $p(-1) = 1 - 2 - 4 + 5 - 6 \neq 0$
 $p(2) = 16 + 16 - 16 - 10 - 6 = 0 \checkmark$

candidates: $\pm 1, \pm 3$
déjà essayé plus haut

$q(3) = 27 + 36 + 12 + 3 \neq 0$
 $q(-3) = -27 + 36 - 12 + 3 = 0 \checkmark$

$\Rightarrow S = \{-3, 2\}$

candidates: $\pm 1, \pm 2, \pm 4, \pm 8$

$p(1) = 1 - 7 + 18 - 20 + 8 = 0 \checkmark$

$\Rightarrow S = \{1, 2\}$

candidates: ± 1

$p(-1) = -35 + 47 - 13 + 1 = 0 \checkmark$

$\Delta = 144 - 4 \cdot 35 \cdot 1 = 4$

$x_{1,2} = \frac{-12 \pm 2}{70} = \begin{cases} -1/7 \\ -1/5 \end{cases}$

$\Rightarrow S = \{-1, -1/5, -1/7\}$

$$d) \quad x^3 + 5x^2 - 8x - 48 = 0$$

1	5	-8	-48
3	3	24	48
1	8	16	0

$$\Rightarrow (x-3)(x^2 + 8x + 16) = 0$$

$$(x-3)(x+4)^2 = 0$$

\downarrow \downarrow
3 -4

candidates : $\pm 1, \pm 2, \pm 3, \pm 4, \dots$
 ± 48

$$p(3) = 0 \checkmark$$

$$\Rightarrow \underline{S = \{-4; 3\}}$$