Ex 1.1.5

a) $\lim\limits_{x\to 2}\frac{e^x-e^2}{x-2}\overset{\text{"}\frac{0}{0}\text{" B-H}}{\overbrace{=}}\lim\limits_{x\to 2}\frac{e^x}{1}=e^2$

(b) $\lim\limits_{x\to 0}\frac{e^x-1}{\sin(x)}\overset{\text{"}\frac{0}{0}\text{" B-H}}{\overbrace{=}}\lim\limits_{x\to 0}\frac{e^x}{\cos(x)}=1$)

c) $\lim\limits_{x\to 0}\frac{x\,e^x}{1-e^x}\overset{\text{"}\frac{0}{0}\text{" B-H}}{\overbrace{=}}\lim\limits_{x\to 0}\frac{(x+1)\,e^x}{-e^x}=-1$

d) $\lim\limits_{x\to 0^+} xe^{1/x} = \text{"}0\cdot e^{\frac{1}{0_+}}\text{"} = \text{"}\overset{\text{f.i.}}{0\cdot\infty}\text{"} = \lim\limits_{x\to 0^+}\frac{e^{1/x}}{1/x}$

$\underset{\text{B.H.}}{\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim\limits_{x\to 0^+}\frac{-\frac{1}{x^2}e^{1/x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0^+} e^{1/x} = e^{+\infty} = \underline{+\infty}$

e) $\underbrace{\lim\limits_{x\to-\infty}\frac{e^x}{2x}}_{=0}+\lim\limits_{x\to-\infty}\frac{e^{-x}}{2x}\overset{\text{"}\frac{\infty}{\infty}\text{" B-H}}{\overbrace{=}}\lim\limits_{x\to-\infty}\frac{-e^{-x}}{2}=-\infty$

f) $\lim\limits_{x\to+\infty}\frac{2\,e^x-1}{e^x+2}\overset{\text{"}\frac{\infty}{\infty}\text{" B-H}}{\overbrace{=}}\lim\limits_{x\to+\infty}\frac{2\,e^x}{e^x}=\lim\limits_{x\to+\infty}2=2$

g) $\lim\limits_{x\to-\infty}(x^2+x)e^x = \text{"}\underset{\text{f.i.}}{\infty\cdot 0}\text{"} = \lim\limits_{x\to-\infty}\frac{x^2+x}{e^{-x}} \underset{\text{B.H.}}{\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim\limits_{x\to-\infty}\frac{2x+1}{-e^{-x}}$

$\underset{\text{B.H.}}{\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim\limits_{x\to-\infty}\frac{2}{e^{-x}} = \frac{2}{\infty} = \underline{0}$

h) $\lim\limits_{x\to+\infty}\frac{e^x}{x^2-2x+3}\overset{\text{"}\frac{\infty}{\infty}\text{" B-H}}{\overbrace{=}}\lim\limits_{x\to+\infty}\frac{e^x}{2x-2}\overset{\text{"}\frac{\infty}{\infty}\text{" B-H}}{\overbrace{=}}\lim\limits_{x\to+\infty}\frac{e^x}{2}=+\infty$

Ex 1.1.9

a) $\lim\limits_{x \to 0} \frac{\ln(x+1)}{x} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim\limits_{x \to 0} \frac{\frac{1}{x+1}}{1} = 1$

$$\left(\text{b)} \lim_{x \to 0} \frac{\ln(\cos(x))}{x^2} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim_{x \to 0} \frac{-\tan(x)}{2x} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim_{x \to 0} \frac{-1-\tan^2(x)}{2} = -\frac{1}{2} \right)$$

c) $\lim\limits_{x \to e} \frac{\ln(x)-1}{x-e} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim\limits_{x \to e} \frac{\frac{1}{x}}{1} = \frac{1}{e}$

d) $\lim\limits_{x \to 2} \frac{\ln(x^2-3)}{2-x} \overbrace{=}^{"\frac{0}{0}" \text{ B-H}} \lim\limits_{x \to 2} \frac{\frac{2x}{x^2-3}}{-1} = -4$

e) $\lim\limits_{x \to -\infty} \frac{\ln(x^2+1)}{x} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim\limits_{x \to -\infty} \frac{\frac{2x}{x^2+1}}{1} = 0$

f) $\lim\limits_{x \to +\infty} \frac{\ln(x)+1}{1-\ln(x)} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim\limits_{x \to +\infty} \frac{\frac{1}{x}}{-\frac{1}{x}} = -1$

g) $\lim\limits_{x \to +\infty} \frac{\ln(x^2)}{\ln^2(x)} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim\limits_{x \to +\infty} \frac{\frac{2}{x}}{2\ln(x) \cdot \frac{1}{x}} = \lim\limits_{x \to +\infty} \frac{1}{\ln(x)} = 0$

h) $\lim\limits_{x \to +\infty} \frac{\ln(x)}{e^x} \overbrace{=}^{"\frac{\infty}{\infty}" \text{ B-H}} \lim\limits_{x \to +\infty} \frac{\frac{1}{x}}{e^x} = 0$