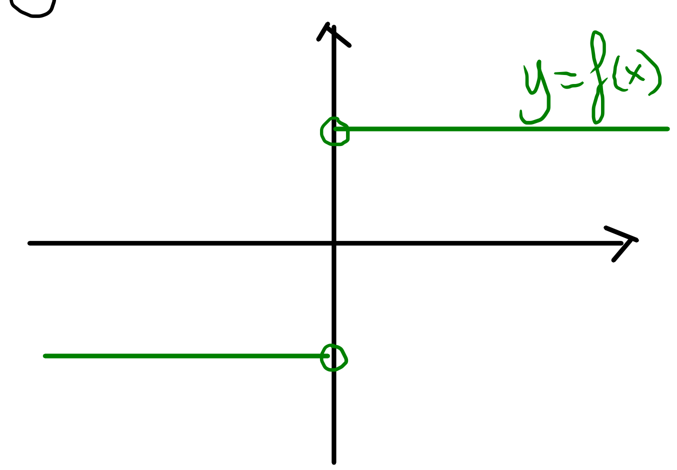
$\lim\limits_{x \to \dots} f(x) = \dots$

Ex 2.6.5

a) $f(x) = \dfrac{|x|}{x} = \begin{cases} \dfrac{-x}{x} & \text{si } x<0 \\ \dfrac{x}{x} & \text{si } x>0 \end{cases} = \begin{cases} -1 & \text{si } x<0 \\ 1 & \text{si } x>0 \end{cases}$

$ED(f) = \mathbb{R}^*$



$\lim\limits_{x \to 0_-} f(x) = -1 \neq \lim\limits_{x \to 0_+} f(x) = 1$

$\Rightarrow \lim\limits_{x \to 0} f(x)$ n'existe pas.

b) $f(x) = \dfrac{x^2 + |x|}{|x|} = \begin{cases} \dfrac{x^2 - x}{-x} & \text{si } x<0 \\ \dfrac{x^2+x}{x} & \text{si } x>0 \end{cases}$

$ED(f) = \mathbb{R}^*$

$\lim\limits_{x \to 0_-} f(x) = \underset{f.i.}{\text{“}\dfrac{0}{0}\text{”}} = \lim\limits_{x \to 0_-} \dfrac{\cancel{x}(x-1)}{-\cancel{x}} = \lim\limits_{x \to 0_-} \dfrac{x-1}{-1} = \dfrac{-1}{-1} = 1$

$= \checkmark$

$\lim\limits_{x \to 0_+} f(x) = \underset{f.i.}{\text{“}\dfrac{0}{0}\text{”}} = \lim\limits_{x \to 0_+} \dfrac{\cancel{x}(x+1)}{\cancel{x}} = \lim\limits_{x \to 0_+} (x+1) = 1$

$\Rightarrow \lim\limits_{x \to 0} f(x) = 1$

2.6.14

a) $$\lim_{x \to -2} \frac{x^2+3x+6}{(x+2)^2} = "\frac{4}{0^2}" = "\frac{4}{0_+}" = +\infty$$

b) $$\lim_{x \to -3} \frac{x^2+2x-15}{x^2+8x+15} = "\frac{-12}{0}" = \begin{cases} x<-3: & "\frac{-12}{0_-}" = +\infty \\ x>-3: & "\frac{-12}{0_+}" = -\infty \end{cases} \quad \infty$$