

3MS - Primitives - Exercices supplémentaires I

$$1) \int x^5 dx \stackrel{\otimes}{=} \underline{\frac{1}{6}x^6 + C}$$

$$\int x^n dx \stackrel{\otimes}{=} \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

$$2) \int 5x^3 dx \stackrel{\otimes}{=} 5 \cdot \frac{1}{4}x^4 + C = \underline{\frac{5}{4}x^4 + C}$$

$$3) \int \frac{1}{2}x^4 dx \stackrel{\otimes}{=} \frac{1}{2} \cdot \frac{1}{5}x^5 + C = \underline{\frac{1}{10}x^5 + C}$$

$$4) \int 3 du = \underline{3u + C}$$

$$5) \int (3t^2 + 5t - 1) dt \stackrel{\otimes}{=} \underline{t^3 + \frac{5}{2}t^2 - t + C}$$

$$6) \int \frac{1}{t^2} dt = \int t^{-2} dt \stackrel{\otimes}{=} \frac{1}{-2+1}t^{-2+1} + C = \frac{1}{-1}t^{-1} + C = \underline{-\frac{1}{t} + C}$$

$$7) \int \sqrt{v} dv = \int v^{1/2} dv \stackrel{\otimes}{=} \frac{1}{\frac{1}{2}+1}v^{\frac{1}{2}+1} + C = \frac{1}{\frac{3}{2}}v^{\frac{3}{2}} + C = \underline{\frac{2}{3}\sqrt{v^3} + C}$$

$$8) \int \sqrt[3]{x} dx = \int x^{1/3} dx \stackrel{\otimes}{=} \frac{1}{\frac{1}{3}+1}x^{\frac{1}{3}+1} + C = \frac{1}{\frac{4}{3}}x^{\frac{4}{3}} + C = \underline{\frac{3}{4}\sqrt[3]{x^4} + C}$$

$$9) \int \frac{5}{\sqrt[4]{x^3}} dx = \int 5x^{-3/4} dx \stackrel{\otimes}{=} 5 \frac{1}{\underbrace{-\frac{3}{4}+1}_{1/4}}x^{-\frac{3}{4}+1} + C = 5 \cdot 4x^{1/4} + C = \underline{20\sqrt[4]{x} + C}$$

$$10) \int \frac{3}{\sqrt{4x+1}} dx = \int 3(4x+1)^{-1/2} dx$$

$$\int u^n \cdot u' dx \stackrel{\otimes}{=} \frac{1}{n+1}u^{n+1} + C, \text{ si } n \neq -1$$

$$\begin{array}{l} u = 4x+1 \\ u' = 4 \end{array} \quad \left| \quad = \int \frac{3 \cdot 4}{4} (4x+1)^{-1/2} dx = \frac{3}{4} \int 4(4x+1)^{-1/2} dx \stackrel{\otimes}{=} \frac{3}{4} \cdot \frac{1}{-\frac{1}{2}+1} (4x+1)^{-\frac{1}{2}+1} + C$$

$$11) \int \underbrace{(3u^2 + 5u - 1)}_u \cdot \underbrace{(6u + 5)}_{u'} du = \frac{3}{2 \cdot 4} \cdot 2(4x+1)^{1/2} + C = \underline{\frac{3}{2}\sqrt{4x+1} + C}$$

$$\stackrel{\otimes}{=} \underline{\frac{1}{4}(3u^2 + 5u - 1)^4 + C}$$

$$12) \int \frac{2x}{(1+x^2)^3} dx = \int \underbrace{2x}_{u'} \cdot \underbrace{(1+x^2)^{-3}}_u dx \stackrel{\textcircled{*}}{=} \frac{1}{-2} (1+x^2)^{-2} + C = \underline{\underline{-\frac{1}{2(1+x^2)^2} + C}}$$

$$13) \int \underbrace{(x^3 + 3x^2 + 2x + 1)}_u \cdot \underbrace{(3x^2 + 6x + 2)}_{u'} dx \stackrel{\textcircled{*}}{=} \frac{1}{1+1} (x^3 + 3x^2 + 2x + 1)^{1+1} + C \\ = \underline{\underline{\frac{1}{2} (x^3 + 3x^2 + 2x + 1)^2 + C}}$$

$$14) \int \frac{\ln^2(x)}{x} dx = \int \frac{1}{x} \cdot \ln^2(x) dx \stackrel{\textcircled{*}}{=} \underline{\underline{\frac{1}{3} \ln^3(x) + C}} \quad \left(\text{Rappel: } (\ln(x))^2 = \ln^2(x) \right)$$

$u = \ln(x)$
 $u' = \frac{1}{x}$

$$15) \int e^x dx \stackrel{\textcircled{*}}{=} \underline{\underline{e^x + C}}$$

$$\int e^u \cdot u' dx \stackrel{\textcircled{*}}{=} e^u + C$$

$$16) \int 2e^{3t} dt = \frac{2}{3} \int 3e^{3t} dt \stackrel{\textcircled{*}}{=} \underline{\underline{\frac{2}{3} e^{3t} + C}}$$

$u = 3t$
 $u' = 3$

$$17) \int 2xe^{-x^2} dx = - \int -2xe^{-x^2} dx \stackrel{\textcircled{*}}{=} \underline{\underline{-e^{-x^2} + C}}$$

$u = -x^2$
 $u' = -2x$

$$18) \int \frac{1}{x} dx = \underline{\underline{\ln(|x|) + C}}$$

$$\int \frac{u'}{u} du \stackrel{\textcircled{*}}{=} \ln(|u|) + C$$

$$19) \int \frac{2}{4x-3} dx = \frac{1}{2} \int \frac{2 \cdot 2}{4x-3} dx \stackrel{\textcircled{*}}{=} \underline{\underline{\frac{1}{2} \ln(|4x-3|) + C}}$$

$u = 4x-3$
 $u' = 4$

$$20) \int \frac{2x+3}{x^2+3x+3} dx \stackrel{\textcircled{*}}{=} \underline{\underline{\ln(|x^2+3x+3|) + C}}$$

$u = x^2 + 3x + 3$
 $u' = 2x + 3$