Ex 1.1.3

a) $\int 2e^x \, dx = 2\int e^x \, dx = \underline{2e^x + c}$

b) $\int e^{2x} \, dx = \frac{1}{2}\int e^{2x} \cdot 2 \, dx = \underline{\frac{1}{2} e^{2x} + c}$

$u = 2x$
$u' = 2$

c) $\int (2 - e^x) \, dx = \underline{2x - e^x + c}$

d) $\int e^{2-x} \, dx = -\int e^{2-x} \cdot (-1) \, dx = \underline{-e^{2-x} + c}$

$u = 2-x$
$u' = -1$

Ex 1.1.4

a) $\displaystyle\int_1^2 e^x dx = e^x \Big|_1^2 = e^2 - e^1 = \underline{e^2 - e}$

b) $\displaystyle\int_1^2 e^{3x-7} dx = \frac{1}{3}\int_1^2 e^{3x-7}\cdot 3dx = \frac{1}{3} e^{3x-7}\Big|_1^2$

$u = 3x-7$
$u' = 3$

$\displaystyle = \frac{1}{3}\left(e^{-1} - e^{-4}\right) = \underline{\frac{1}{3}\left(\frac{1}{e} - \frac{1}{e^4}\right)} = \frac{1}{3}\,\frac{e^3-1}{e^4}$

$\displaystyle = \underline{\frac{e^3-1}{3e^4}}$

c) $\displaystyle\int_0^2 xe^{x^2} dx = \frac{1}{2}\int_0^2 2x\, e^{x^2} dx = \frac{1}{2} e^{x^2}\Big|_0^2$

$u = x^2$
$u' = 2x$

$\displaystyle = \frac{1}{2}\left(e^4 - e^0\right) = \underline{\frac{1}{2}\left(e^4 - 1\right)} = \underline{\frac{e^4-1}{2}}$

d) $\displaystyle\int_1^2 \frac{1}{\sqrt{x}\,e^{\sqrt{x}}} dx = -2\int_1^2 -\frac{1}{2}\cdot\frac{1}{\sqrt{x}}\cdot e^{-\sqrt{x}} dx = -2e^{-\sqrt{x}}\Big|_1^2$

si $u = \sqrt{x}$
$\displaystyle u' = \frac{1}{2\sqrt{x}}$

si $u = -\sqrt{x}$
$\displaystyle u' = -\frac{1}{2\sqrt{x}} = -\frac{1}{2}\cdot\frac{1}{\sqrt{x}}$

$= -2e^{-\sqrt{2}} + 2e^{-1}$

$\displaystyle = \frac{-2}{e^{\sqrt{2}}} + \frac{2}{e} = \underline{-2\left(\frac{1}{e^{\sqrt{2}}} - \frac{1}{e}\right)}$

Ex 1.1.7

a) $\int \frac{1}{x+1}\,dx = \ln(|x+1|) + C$

b) $\int \frac{1}{3x+2}\,dx = \frac{1}{3}\int \frac{3}{3x+2}\,dx = \frac{1}{3}\ln(|3x+2|) + C = \ln\left(\sqrt[3]{|3x+2|}\right) + C$

$u = 3x+2$
$u' = 3$

c) $\int \frac{x-1}{x^2-2x+4}\,dx = \frac{1}{2}\int \frac{2(x-1)}{x^2-2x+4}\,dx = \frac{1}{2}\ln(|x^2-2x+4|) + C$

$u = x^2-2x+4$
$u' = 2x-2 = 2(x-1)$

$= \ln\left(\sqrt{|x^2-2x+4|}\right) + C$

d) $\int \left(x^2+x+1+\frac{3}{5x-1}\right)dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c_1 + \frac{3}{5}\int \frac{5}{5x-1}\,dx$

$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \frac{3}{5}\ln(|5x-1|) + c$

e) $\int \frac{x^2+2x-2}{x-1}\,dx$

$\deg(N) \geq \deg(D) \Rightarrow$ div. euclidienne

$$\begin{array}{r|l} x^2+2x-2 & x-1 \\ \cline{2-2} -x^2+x & x+3 \\ \hline 3x-2 & \\ -3x+3 & \\ \hline 1 & \end{array}$$

$\Rightarrow f(x) = x+3+\frac{1}{x-1}$

$= \int \left(x+3+\frac{1}{x-1}\right)dx = \frac{1}{2}x^2 + 3x + \ln(|x-1|) + C$

Ex 1.1.8

a) $\displaystyle\int_2^5 \frac{dx}{x} = \ln(|x|)\Big|_2^5 = \ln(5) - \ln(2) = \ln\left(\frac{5}{2}\right)$

b) $\displaystyle\int_{-1}^{-3} \frac{dx}{x} = \ln(|x|)\Big|_{-1}^{-3} = \ln(3) - \ln(1) = \ln(3)$

c) $\displaystyle\int_{-1}^{4} \frac{dx}{x} = \lim_{\varepsilon \to 0}\left(\int_{-1}^{0-\varepsilon} \frac{dx}{x} + \int_{0+\varepsilon}^{4} \frac{dx}{x}\right) = \lim_{\varepsilon \to 0}\left(\ln(|x|)\Big|_{-1}^{0-\varepsilon} + \ln(|x|)\Big|_{0+\varepsilon}^{4}\right)$

(⚠ pas défini en 0)

$\displaystyle= \lim_{\varepsilon \to 0}\left(\ln(|0-\varepsilon|) - \ln(1) + \ln(4) - \ln(|0+\varepsilon|)\right)$

$= \ln(4)$

d) $\displaystyle\int_1^4 \frac{dx}{2x+3} = \frac{1}{2}\int_1^4 \frac{2}{2x+3}dx = \frac{1}{2}\ln(|2x+3|)\Big|_1^4$

$u = 2x+3$
$u' = 2$

$\displaystyle= \frac{1}{2}\left(\ln(11) - \ln(5)\right) = \frac{1}{2}\ln\left(\frac{11}{5}\right) = \ln\left(\sqrt{\frac{11}{5}}\right)$

e) $\displaystyle\int_2^6 \frac{8x^3+19x^2+15x+4}{x^2+2x+1}dx$

$\deg(N) \geq \deg(D) \Rightarrow$ div. eucl.

$$\begin{array}{r|l} 8x^3+19x^2+15x+4 & x^2+2x+1 \\ -8x^3-16x^2-8x & 8x+3 \\ \hline 3x^2+7x+4 & \\ -3x^2-6x-3 & \\ \hline x+1 & \end{array}$$

$\displaystyle= \int_2^6 \left(8x+3 + \frac{x+1}{x^2+2x+1}\right)dx$

$\displaystyle= \int_2^6 (8x+3)dx + \frac{1}{2}\int_2^6 \frac{2(x+1)}{x^2+2x+1}dx$

$\displaystyle= 4x^2+3x\Big|_2^6 + \frac{1}{2}\ln(|x^2+2x+1|)\Big|_2^6 = 162-22+\frac{1}{2}\ln(49) - \frac{1}{2}\ln(9)$

$\displaystyle= 140 + \ln(7) - \ln(3) = 140 + \ln\left(\frac{7}{3}\right)$