

Ex 1.3.7Rem: 3 coords, on travaille dans l'espace ($\mathbb{R}^3$)

$$a) \vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} 8 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 8-5 \\ 0-2 \\ 5+3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}}}$$

$$b) \vec{BD} = \vec{OD} - \vec{OB} = \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 4-8 \\ -6-0 \\ 3-5 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -4 \\ -6 \\ -2 \end{pmatrix}}}$$

$$c) \vec{CA} = \vec{OA} - \vec{OC} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 5+2 \\ 2+4 \\ -3-1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 7 \\ 6 \\ -4 \end{pmatrix}}}$$

$$d) \vec{AD} + \vec{CB} = \vec{OD} - \vec{OA} + \vec{OB} - \vec{OC} = \begin{pmatrix} 4-5 \\ -6-2 \\ 3+3 \end{pmatrix} + \begin{pmatrix} 8+2 \\ 0+4 \\ 5-1 \end{pmatrix} = \begin{pmatrix} -1 \\ -8 \\ 6 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \\ 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 9 \\ -4 \\ 10 \end{pmatrix}}}$$

$\vec{DA} = -\vec{AD}$

$$e) \vec{BC} - \vec{AC} + \vec{DB} = \vec{DB} + \vec{BC} + \vec{CA} = \vec{DA} = \underline{\underline{\begin{pmatrix} 1 \\ 8 \\ -6 \end{pmatrix}}}$$

↑
Chasles

Variante :

$$= \begin{pmatrix} -2-8 \\ -4-0 \\ 1-5 \end{pmatrix} - \begin{pmatrix} -2-5 \\ -4-2 \\ 1+3 \end{pmatrix} + \begin{pmatrix} 8-4 \\ 0+6 \\ 5-3 \end{pmatrix} = \begin{pmatrix} -10 \\ -4 \\ -4 \end{pmatrix} - \begin{pmatrix} -7 \\ -6 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ -6 \end{pmatrix}$$

$$f) 4\vec{CD} - 3(\underbrace{\vec{CA} + \vec{BC}}_{\vec{BC} + \vec{CA}}) = 4\vec{CD} - 3\vec{BA} = 4\vec{CD} + 3\vec{AB}$$

$$= 4 \begin{pmatrix} 4+2 \\ -6+4 \\ 3-1 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix} = 4 \begin{pmatrix} 6 \\ -2 \\ 2 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}$$

↑
a)

$$= \begin{pmatrix} 24+9 \\ -8-6 \\ 8+24 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 33 \\ -14 \\ 32 \end{pmatrix}}}$$

Ex 1.3.8

$A(1;1)$ $B(10;5)$ $C(4;12)$

a) ABCD parallélogramme $\Leftrightarrow \vec{AB} = \vec{DC}$ et $\vec{AB} = \begin{pmatrix} 10-1 \\ 5-1 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$

On cherche D :

$$\vec{OD} = \vec{OC} + \vec{CD} = \vec{OC} - \vec{DC} = \vec{OC} - \vec{AB} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} - \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}$$

$\Rightarrow \underline{D(-5;8)}$

Variante : on pose $D(d_1, d_2)$

$$\vec{AB} = \vec{DC} \Leftrightarrow \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} 4-d_1 \\ 12-d_2 \end{pmatrix} \Leftrightarrow \begin{cases} 9 = 4-d_1 \\ 4 = 12-d_2 \end{cases} \Leftrightarrow \begin{cases} d_1 = -5 \\ d_2 = 8 \end{cases}$$

b) ABDC // gramme $\Leftrightarrow \vec{AB} = \vec{CD}$

On cherche D :

$$\vec{OD} = \vec{OC} + \vec{CD} = \vec{OC} + \vec{AB} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} + \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 16 \end{pmatrix}$$

$\Rightarrow \underline{D(13;16)}$

Variante : on pose $D(d_1, d_2)$

$$\vec{AB} = \vec{CD} \Leftrightarrow \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} d_1-4 \\ d_2-12 \end{pmatrix} \Leftrightarrow \begin{cases} 9 = d_1-4 \\ 4 = d_2-12 \end{cases} \Leftrightarrow \begin{cases} d_1 = 13 \\ d_2 = 16 \end{cases}$$

Ex 1.3.11 $M(13; -22; 2)$ $N(-5; -10; 26)$ et $P(-38; 12; 60)$

Rem: 3 coords, on travaille dans l'espace ($\mathbb{R}^3$)

M, N et P sont alignés $\Leftrightarrow \vec{MN} \sim \vec{NP}$

$$\vec{MN} = \begin{pmatrix} -5 - 13 \\ -10 + 22 \\ 26 - 2 \end{pmatrix} = \begin{pmatrix} -18 \\ 12 \\ 24 \end{pmatrix}$$

$$\vec{NP} = \begin{pmatrix} -38 + 5 \\ 12 + 10 \\ 60 - 26 \end{pmatrix} = \begin{pmatrix} -33 \\ 22 \\ 34 \end{pmatrix}$$

les vecteurs sont colinéaires s'il existe $k \in \mathbb{R}$ tq $\vec{MN} = k \cdot \vec{NP}$

$$-18 = -33k \quad \Leftrightarrow \quad k = \frac{18}{33} = \frac{6}{11}$$

$$12 = 22k \quad \Leftrightarrow \quad k = \frac{12}{22} = \frac{6}{11}$$

$$24 = 34k \quad \Leftrightarrow \quad k = \frac{24}{34} = \frac{12}{17}$$

$\swarrow$
 $\neq$ pas égaux

$$\Rightarrow \vec{MN} \neq k \cdot \vec{NP} \quad \Leftrightarrow \quad \vec{MN} \not\sim \vec{NP}$$

$\Leftrightarrow$ M, N et P ne sont pas alignés.

Ex 1.3.12

A, B et C alignés $\Leftrightarrow \overrightarrow{AB} \sim \overrightarrow{BC}$

a) $A(1;2)$ $B(-3;3)$ et $C(k;1)$

$$\overrightarrow{AB} = \begin{pmatrix} -3-1 \\ 3-2 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \quad \text{et} \quad \overrightarrow{BC} = \begin{pmatrix} k+3 \\ 1-3 \end{pmatrix} = \begin{pmatrix} k+3 \\ -2 \end{pmatrix}$$

$$\overrightarrow{AB} \sim \overrightarrow{BC} \Leftrightarrow \det(\overrightarrow{AB}, \overrightarrow{BC}) = 0$$

$$\Leftrightarrow \begin{vmatrix} -4 & k+3 \\ 1 & -2 \end{vmatrix} = 0$$

$$\Leftrightarrow -4 \cdot (-2) - 1 \cdot (k+3) = 0$$

$$\Leftrightarrow 8 - k - 3 = 0 \Leftrightarrow \underline{k = 5} \Rightarrow \underline{C(5;1)}$$

b) $A(2;k)$, $B(7k-29;5)$ et $C(-4;2)$

$$\overrightarrow{AB} = \begin{pmatrix} 7k-29-2 \\ 5-k \end{pmatrix} = \begin{pmatrix} 7k-31 \\ 5-k \end{pmatrix} \quad \text{et} \quad \overrightarrow{BC} = \begin{pmatrix} -4-(7k-29) \\ 2-5 \end{pmatrix} = \begin{pmatrix} -7k+25 \\ -3 \end{pmatrix}$$

$$\overrightarrow{AB} \sim \overrightarrow{BC} \Leftrightarrow \det(\overrightarrow{AB}, \overrightarrow{BC}) = 0$$

$$\Leftrightarrow \begin{vmatrix} 7k-31 & -7k+25 \\ 5-k & -3 \end{vmatrix} = 0$$

$$\Leftrightarrow (7k-31) \cdot (-3) - (5-k)(-7k+25) = 0$$

$$\Leftrightarrow -21k + 93 - (-35k + 125 + 7k^2 - 25k) = 0$$

$$\Leftrightarrow -21k + 93 + 35k - 125 - 7k^2 + 25k = 0$$

$$\Leftrightarrow -7k^2 + 39k - 32 = 0$$

$$\Delta = 39^2 - 4 \cdot (-7) \cdot (-32) = 625 \Rightarrow k_{1,2} = \frac{-39 \pm 25}{-14} = \begin{cases} \frac{-14}{-14} = \underline{1} \\ \frac{-64}{-14} = \underline{\frac{32}{7}} \end{cases}$$

Ex 1.3.14

$A(7; -3)$ $B(23; -6)$ et $C \in O_x \Leftrightarrow C(x; 0)$
← "appartient à"

$$\vec{AB} = \begin{pmatrix} 23 - 7 \\ -6 + 3 \end{pmatrix} = \begin{pmatrix} 16 \\ -3 \end{pmatrix} \quad \text{et} \quad \vec{AC} = \begin{pmatrix} x - 7 \\ 0 + 3 \end{pmatrix} = \begin{pmatrix} x - 7 \\ 3 \end{pmatrix}$$

A, B et C alignés $\Leftrightarrow \vec{AB} \sim \vec{AC}$

$$\Leftrightarrow \det(\vec{AB}; \vec{AC}) = 0$$

$$\Leftrightarrow \begin{vmatrix} 16 & x-7 \\ -3 & 3 \end{vmatrix} = 0$$

$$\Leftrightarrow 16 \cdot 3 - (-3)(x-7) = 0$$

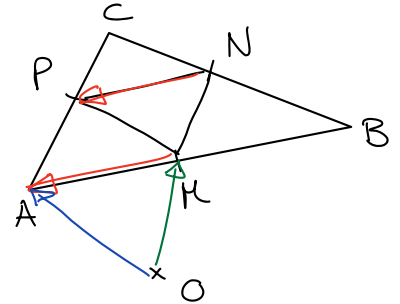
$$\Leftrightarrow 48 + 3(x-7) = 0$$

$$\Leftrightarrow 48 + 3x - 21 = 0$$

$$\Leftrightarrow 3x = -27 \Leftrightarrow x = -9 \Rightarrow \underline{C(-9; 0)}$$

Ex 1.3.18

$$* \quad \vec{OA} = \vec{OM} + \vec{MA} = \vec{OM} + \vec{NP}$$



$$\text{Calculons } \vec{NP} = \begin{pmatrix} -2 - (-1) \\ 2 - 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\Rightarrow \vec{OA} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \Rightarrow \underline{A(1; -3)}$$

$$* \quad \vec{OB} = \vec{OM} + \vec{MB} = \vec{OM} + \vec{PN} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow \underline{B(3; 1)}$$

$$* \quad \vec{OC} = \vec{OP} + \vec{PC} = \vec{OP} + \vec{MN}$$

$$\text{Calculons } \vec{MN} = \begin{pmatrix} -1 - 2 \\ 4 - (-1) \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

$$\Rightarrow \vec{OC} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \Rightarrow \underline{C(-5; 7)}$$

Ex 1.3.19 $A(-4; 2)$ $B(1; 3)$ $C(2; 5)$

$$M_{AB}\left(\frac{-4+1}{2}; \frac{2+3}{2}\right) = \underline{M_{AB}\left(-\frac{3}{2}; \frac{5}{2}\right)}$$

$$M_{AC}\left(\frac{-4+2}{2}; \frac{2+5}{2}\right) = \underline{M_{AC}\left(-1; \frac{7}{2}\right)}$$

$$M_{BC}\left(\frac{1+2}{2}; \frac{3+5}{2}\right) = \underline{M_{BC}\left(\frac{3}{2}; 4\right)}$$

$$G_{ABC}\left(\frac{-4+1+2}{3}; \frac{2+3+5}{3}\right) = \underline{G_{ABC}\left(-\frac{1}{3}; \frac{10}{3}\right)}$$

Ex 1.3.21 $A(2;-1)$ $B(0;3)$

a) On cherche C tq. $O(0;0)$ est le centre de gravité du $\triangle ABC$

On peut poser $C(c_1; c_2)$

$$\left(\frac{2+0+c_1}{3}, \frac{-1+3+c_2}{3} \right) = (0;0)$$

$$\Leftrightarrow \begin{cases} \frac{2+c_1}{3} = 0 \\ \frac{2+c_2}{3} = 0 \end{cases} \Leftrightarrow \begin{cases} 2+c_1 = 0 \\ 2+c_2 = 0 \end{cases} \Leftrightarrow \begin{cases} c_1 = -2 \\ c_2 = -2 \end{cases} \Rightarrow \underline{C(-2;-2)}$$

b) On cherche D tq. $ABCD$ // - gramme $\Leftrightarrow \vec{AB} = \vec{DC}$

$$\vec{AB} = \begin{pmatrix} 0-2 \\ 3+1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$\vec{OD} = \vec{OC} + \vec{CD} = \vec{OC} - \vec{DC} = \vec{OC} - \vec{AB} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} \Rightarrow \underline{D(0;-6)}$$

Variante: on pose $D(d_1, d_2)$

$$\vec{AB} = \vec{DC} \Leftrightarrow \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2-d_1 \\ -2-d_2 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} -2-d_1 = -2 \\ -2-d_2 = 4 \end{cases} \Leftrightarrow \begin{cases} d_1 = 0 \\ d_2 = -6 \end{cases}$$