Ex 1.1.1

a) $\underline{3+i}$ b) $\underline{0}$ c) $\underline{-1+7i}$ d) $\underline{-7i}$ e) $\underline{17+i}$

f) $(9+5i)(2-7i) = 18-63i+10-35i^2 = \underline{53-53i}$

g) $(3+2i)(3-2i) = 9+4 = \underline{13}$

h) $(3-4i)^2 = 9-24i+16i^2 = \underline{-7-24i}$

i) $(1+i)^4 = (1+2i+i^2)^2 = (2i)^2 = 4i^2 = \underline{-4}$

j) $\frac{1}{i}\cdot\frac{-i}{-i} = \frac{-i}{-i^2} = \frac{-i}{1} = \underline{-i}$ variante : $\frac{1}{i}\cdot\frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$

k) $\frac{1}{2+3i}\cdot\frac{2-3i}{2-3i} = \frac{2-3i}{4+9} = \underline{\frac{2}{13}-\frac{3}{13}i}$

l) $\frac{1+i}{1-i}\cdot\frac{1+i}{1+i} = \frac{1+2i+i^2}{1+1} = \frac{2i}{i} = \underline{2}$

m) $\frac{5+3i}{2+4i}\cdot\frac{2-4i}{2-4i} = \frac{10-20i+6i+12}{4+16} = \frac{22-14i}{20} = \underline{\frac{11}{10}-\frac{7}{10}i}$

n) $\left(\frac{63+16i}{4+3i}\right)^2 = \left(\frac{63+16i}{4+3i}\cdot\frac{4-3i}{4-3i}\right)^2 = \left(\frac{252-189i+64i+48}{16+9}\right)^2$

$= \left(\frac{300-125i}{25}\right)^2 = (12-5i)^2 = 144-120i-25 = \underline{119-120i}$

variante $= \frac{3969+2016i-256}{16+24i-9} = \frac{3713+2016i}{7+24i}\cdot\frac{7-24i}{7-24i}$

$= \frac{25991-89112i+14112i+48384}{49+576} = \frac{74375-75000i}{625} = 119-120i$

Ex 1.1.2

$i^0 = 1, \quad i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1, \quad i^5 = i, \quad i^6 = -1, \quad \dots$

$\cdot i \quad \cdot i \quad \cdot i \quad \cdot i \quad \cdot i \quad \cdot i$

$\Rightarrow \begin{cases} i^{4n} = 1 \\ i^{4n+1} = i \\ i^{4n+2} = -1 \\ i^{4n+3} = -i \end{cases}$ avec $n \in \mathbb{N}$

Ex 1.1.3

a) $2z - 3 + i = 0$

$z = \frac{3-i}{2} \Rightarrow S = \left\{ \frac{3}{2} - \frac{1}{2}i \right\}$

b) $(1-4i)z = 6-7i$

$z = \frac{6-7i}{1-4i} \cdot \frac{1+4i}{1+4i} = \frac{6+24i-7i+28}{1+16} = \frac{34+17i}{17} = 2+i$

$\Rightarrow S = \{2+i\}$

c) $(1+2i)z = (5-i)z + 7 + 26i$

$(1+2i)z - (5-i)z = 7 + 26i$

$(-4+3i)z = 7+26i$

⚠ possible mais ce n'est pas le conjugué de $-4+3i$

$z = \frac{7+26i}{-4+3i} \cdot \frac{4+3i}{4+3i} = \frac{28+21i+104i-78}{-16-9}$

$= \frac{-50+125i}{-25} = 2-5i$

au $z = \frac{7+26i}{-4+3i} \cdot \frac{-4-3i}{-4-3i} = \frac{-28-21i-104i+78}{16+9}$

$= \frac{50-125i}{25} = 2-5i$

$\Rightarrow S = \{2-5i\}$

Ex 1.1.4

$$\begin{cases} (2+i)z + (2-i)w = 7-4i & \mid \cdot i \\ (1+i)z - i\,w = 2+i & \mid \cdot (2-i) \end{cases}$$

$$\Leftrightarrow \begin{cases} (2i-1)z + (2i+1)w = 7i+4 \\ \underbrace{(1+i)(2-i)}_{2-i+2i+1}z - (2i+1)w = 4+1 \end{cases} \Rightarrow + \begin{array}{l} (-1+2i)z + (2i+1)w = 4+7i \\ (3+i)z - (2i+1)w = 5 \\ \hline (2+3i)z = 9+7i \end{array}$$

$\Rightarrow z = \frac{9+7i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{18-27i+14i+21}{4+9} = \frac{39-13i}{13} = 3-i$

Ex 1.1.4 (suite)

$$\Rightarrow (1+i)(3-i) - iw = 2+i$$
$$-iw = 2+i-(3-i+3i+1) = -2-i$$
$$w = \frac{2+i}{i}\cdot\frac{i}{i} = \frac{-1+2i}{-1} = 1-2i$$

$$\Rightarrow S = \{(1-2i\,;\,3-i)\}$$

Ex 1.1.5

a) $\overline{z_1} = 5+4i$ b) $-8+i$ c) $2-3i$ d) $5+\frac{3}{2}i$

Ex 1.1.6

$$(7-8i)\overline{(8-7i)} - \overline{(4+3i)}(4-2i)$$
$$= (7-8i)(8+7i) - (4-3i)(4-2i)$$
$$= 56+49i-64i+56 - (16-8i-12i-6) = 102+5i$$

Ex 1.1.7

On pose $z = x+yi$

Rappel : $a+bi = c+di \Leftrightarrow a=c$ et $b=d$

a) $8z+5\overline{z} = 4+3i$

$\Leftrightarrow 8(x+yi)+5(x-yi) = 4+3i$

$\Leftrightarrow 13x+3yi = 4+3i$

$\Leftrightarrow \begin{cases} 13x = 4 \\ 3y = 3 \end{cases} \Leftrightarrow \begin{cases} x = \frac{4}{13} \\ y = 1 \end{cases} \Leftrightarrow z = \frac{4}{13}+i \Rightarrow S = \left\{\frac{4}{13}+i\right\}$

b) $z^2+2\overline{z}+5 = 0$

$\Leftrightarrow (x+yi)^2+2(x-yi)+5 = 0$

$\Leftrightarrow x^2+2xyi-y^2+2x-2yi+5 = 0$

$\Leftrightarrow \begin{cases} x^2-y^2+2x+5 = 0 \\ 2xy-2y = 0 \quad |:2 \end{cases} \Leftrightarrow \begin{cases} x^2-y^2+2x+5 = 0 \\ y(x-1) = 0 \end{cases} \Leftrightarrow x=1$ ou $y=0$

$\Leftrightarrow \begin{cases} x=1 \\ 1-y^2+2+5=0 \end{cases}$ ou $\begin{cases} y=0 \\ \underbrace{x^2+2x+5=0} \end{cases}$

$\Delta = 4-20<0$ impossible car $x\in\mathbb{R}$

$\Leftrightarrow \begin{cases} x=1 \\ y^2=8 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ y=\pm\sqrt{8} \end{cases}$

$$\Rightarrow S = \{1+2\sqrt{2}i\,;\,1-2\sqrt{2}i\}$$

c) $2\overset{Im}{\mathcal{I}}(\bar{z}+1) + 2i\overset{Re}{\mathcal{R}}(-z+2) = -1-12i$ $\quad (z = x+yi \quad \bar{z} = x-yi)$

$\Leftrightarrow 2\mathcal{I}(x+1-yi) + 2i\mathcal{R}(-x+2-yi) = -1-12i$

$\Leftrightarrow 2\cdot(-y) + 2i\cdot(-x+2) = -1-12i$

$\Leftrightarrow -2y - 2xi + 4i = -1-12i$

$\Leftrightarrow \begin{cases} -2y = -1 \\ -2x+4 = -12 \end{cases} \Leftrightarrow \begin{cases} y = \frac{1}{2} \\ x = 8 \end{cases} \Rightarrow S = \{8 + \frac{1}{2}i\}$

Ex 1.1.8

1) $z + \bar{z} = 2\mathcal{R}(z)$ On pose $z = x+yi$

$z+\bar{z} = x+yi + x-yi = 2x = 2\mathcal{R}(z)$ #

$\Rightarrow \mathcal{R}(z) = \frac{z+\bar{z}}{2}$

2) $z - \bar{z} = 2\mathcal{I}(z)i$

$z-\bar{z} = x+yi - x+yi = 2yi = 2\mathcal{I}(z)i$ #

$\Rightarrow \mathcal{I}(z) = \frac{z-\bar{z}}{2}$

3) $z\bar{z} = \mathcal{R}(z)^2 + \mathcal{I}(z)^2$

$(x+yi)(x-yi) = x^2+y^2 = \mathcal{R}(z)^2 + \mathcal{I}(z)^2$ #