

Ex 1.1.1

a) $3+i$ b) 0 c) $-1+7i$ d) $-7i$ e) $17+i$

f) $(9+5i)(2-7i) = 18-63i+10-35i^2 = \underline{53-53i}$

g) $(3+2i)(3-2i) = 9+4 = \underline{13}$

h) $(3-4i)^2 = 9-24i+16i^2 = \underline{-7-24i}$

i) $(1+i)^4 = (1+2i+i^2)^2 = (2i)^2 = 4i^2 = \underline{-4}$

j) $\frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{-i^2} = \frac{-i}{1} = \underline{-i}$ variante : $\frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = \underline{-i}$

b) $\frac{1}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i}{4+9} = \underline{\frac{2}{13} - \frac{3}{13}i}$

l) $\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1+2i+i^2}{1+1} = \frac{2i}{1} = \underline{2}$

m) $\frac{5+3i}{2+4i} \cdot \frac{2-4i}{2-4i} = \frac{10-20i+6i+12}{4+16} = \frac{22-14i}{20} = \underline{\frac{11}{10} - \frac{7}{10}i}$

n) $\left(\frac{63+16i}{4+3i}\right)^2 = \left(\frac{63+16i}{4+3i} \cdot \frac{4-3i}{4-3i}\right)^2 = \left(\frac{252-189i+64i+48}{16+9}\right)^2$
 $= \left(\frac{300-125i}{25}\right)^2 = (12-5i)^2 = 144-120i-25 = \underline{119-120i}$

variante $= \frac{3969 + 2016i - 256}{16+24i-9} = \frac{3713 + 2016i}{7+24i} \cdot \frac{7-24i}{7-24i}$
 $= \frac{25931 - 8912i + 14112i + 48384}{49+576} = \frac{74375 - 75000i}{625} = \underline{119-120i}$

Ex 1.1.2

$i^0 = 1$ $\xrightarrow{\cdot i}$ $i^1 = i$ $\xrightarrow{\cdot i}$ $i^2 = -1$ $\xrightarrow{\cdot i}$ $i^3 = -i$ $\xrightarrow{\cdot i}$ $i^4 = 1$ $\xrightarrow{\cdot i}$ $i^5 = i$ $\xrightarrow{\cdot i}$ $i^6 = -1$ $\xrightarrow{\cdot i}$ $i^7 = -i$ $\xrightarrow{\cdot i}$ $i^8 = 1$ $\xrightarrow{\cdot i}$ $i^9 = i$ $\xrightarrow{\cdot i}$ $i^{10} = -1$ $\xrightarrow{\cdot i}$ $i^{11} = -i$ $\xrightarrow{\cdot i}$ $i^{12} = 1$ $\xrightarrow{\cdot i}$ $i^{13} = i$ $\xrightarrow{\cdot i}$ $i^{14} = -1$ $\xrightarrow{\cdot i}$ $i^{15} = -i$ $\xrightarrow{\cdot i}$ $i^{16} = 1$ $\xrightarrow{\cdot i}$ $i^{17} = i$ $\xrightarrow{\cdot i}$ $i^{18} = -1$ $\xrightarrow{\cdot i}$ $i^{19} = -i$ $\xrightarrow{\cdot i}$ $i^{20} = 1$ $\xrightarrow{\cdot i}$ $i^{21} = i$ $\xrightarrow{\cdot i}$ $i^{22} = -1$ $\xrightarrow{\cdot i}$ $i^{23} = -i$ $\xrightarrow{\cdot i}$ $i^{24} = 1$ $\xrightarrow{\cdot i}$ $i^{25} = i$ $\xrightarrow{\cdot i}$ $i^{26} = -1$ $\xrightarrow{\cdot i}$ $i^{27} = -i$ $\xrightarrow{\cdot i}$ $i^{28} = 1$ $\xrightarrow{\cdot i}$ $i^{29} = i$ $\xrightarrow{\cdot i}$ $i^{30} = -1$ $\xrightarrow{\cdot i}$ $i^{31} = -i$ $\xrightarrow{\cdot i}$ $i^{32} = 1$ $\xrightarrow{\cdot i}$ $i^{33} = i$ $\xrightarrow{\cdot i}$ $i^{34} = -1$ $\xrightarrow{\cdot i}$ $i^{35} = -i$ $\xrightarrow{\cdot i}$ $i^{36} = 1$ $\xrightarrow{\cdot i}$ $i^{37} = i$ $\xrightarrow{\cdot i}$ $i^{38} = -1$ $\xrightarrow{\cdot i}$ $i^{39} = -i$ $\xrightarrow{\cdot i}$ $i^{40} = 1$ $\xrightarrow{\cdot i}$ $i^{41} = i$ $\xrightarrow{\cdot i}$ $i^{42} = -1$ $\xrightarrow{\cdot i}$ $i^{43} = -i$ $\xrightarrow{\cdot i}$ $i^{44} = 1$ $\xrightarrow{\cdot i}$ $i^{45} = i$ $\xrightarrow{\cdot i}$ $i^{46} = -1$ $\xrightarrow{\cdot i}$ $i^{47} = -i$ $\xrightarrow{\cdot i}$ $i^{48} = 1$ $\xrightarrow{\cdot i}$ $i^{49} = i$ $\xrightarrow{\cdot i}$ $i^{50} = -1$ $\xrightarrow{\cdot i}$ $i^{51} = -i$ $\xrightarrow{\cdot i}$ $i^{52} = 1$ $\xrightarrow{\cdot i}$ $i^{53} = i$ $\xrightarrow{\cdot i}$ $i^{54} = -1$ $\xrightarrow{\cdot i}$ $i^{55} = -i$ $\xrightarrow{\cdot i}$ $i^{56} = 1$ $\xrightarrow{\cdot i}$ $i^{57} = i$ $\xrightarrow{\cdot i}$ $i^{58} = -1$ $\xrightarrow{\cdot i}$ $i^{59} = -i$ $\xrightarrow{\cdot i}$ $i^{60} = 1$ $\xrightarrow{\cdot i}$ $i^{61} = i$ $\xrightarrow{\cdot i}$ $i^{62} = -1$ $\xrightarrow{\cdot i}$ $i^{63} = -i$ $\xrightarrow{\cdot i}$ $i^{64} = 1$ $\xrightarrow{\cdot i}$ $i^{65} = i$ $\xrightarrow{\cdot i}$ $i^{66} = -1$ $\xrightarrow{\cdot i}$ $i^{67} = -i$ $\xrightarrow{\cdot i}$ $i^{68} = 1$ $\xrightarrow{\cdot i}$ $i^{69} = i$ $\xrightarrow{\cdot i}$ $i^{70} = -1$ $\xrightarrow{\cdot i}$ $i^{71} = -i$ $\xrightarrow{\cdot i}$ $i^{72} = 1$ $\xrightarrow{\cdot i}$ $i^{73} = i$ $\xrightarrow{\cdot i}$ $i^{74} = -1$ $\xrightarrow{\cdot i}$ $i^{75} = -i$ $\xrightarrow{\cdot i}$ $i^{76} = 1$ $\xrightarrow{\cdot i}$ $i^{77} = i$ $\xrightarrow{\cdot i}$ $i^{78} = -1$ $\xrightarrow{\cdot i}$ $i^{79} = -i$ $\xrightarrow{\cdot i}$ $i^{80} = 1$ $\xrightarrow{\cdot i}$ $i^{81} = i$ $\xrightarrow{\cdot i}$ $i^{82} = -1$ $\xrightarrow{\cdot i}$ $i^{83} = -i$ $\xrightarrow{\cdot i}$ $i^{84} = 1$ $\xrightarrow{\cdot i}$ $i^{85} = i$ $\xrightarrow{\cdot i}$ $i^{86} = -1$ $\xrightarrow{\cdot i}$ $i^{87} = -i$ $\xrightarrow{\cdot i}$ $i^{88} = 1$ $\xrightarrow{\cdot i}$ $i^{89} = i$ $\xrightarrow{\cdot i}$ $i^{90} = -1$ $\xrightarrow{\cdot i}$ $i^{91} = -i$ $\xrightarrow{\cdot i}$ $i^{92} = 1$ $\xrightarrow{\cdot i}$ $i^{93} = i$ $\xrightarrow{\cdot i}$ $i^{94} = -1$ $\xrightarrow{\cdot i}$ $i^{95} = -i$ $\xrightarrow{\cdot i}$ $i^{96} = 1$ $\xrightarrow{\cdot i}$ $i^{97} = i$ $\xrightarrow{\cdot i}$ $i^{98} = -1$ $\xrightarrow{\cdot i}$ $i^{99} = -i$ $\xrightarrow{\cdot i}$ $i^{100} = 1$

$\Rightarrow \begin{cases} i^{4n} = 1 \\ i^{4n+1} = i \\ i^{4n+2} = -1 \\ i^{4n+3} = -i \end{cases}$ avec $n \in \mathbb{N}$

Ex 1.1.3

a) $2z - 3 + i = 0$

$$z = \frac{3-i}{2} \Rightarrow S = \left\{ \frac{3-i}{2} \right\}$$

b) $(1-4i)z = 6-7i$

$$z = \frac{6-7i}{1-4i} \cdot \frac{1+4i}{1+4i} = \frac{6+24i-7i+28}{1+16} = \frac{34+17i}{17} = 2+i$$

$$\Rightarrow S = \{2+i\}$$

c) $(1+2i)z = (5-i)z + 7 + 26i$

$$(1+2i)z - (5-i)z = 7 + 26i$$

$$(-4+3i)z = 7 + 26i$$

$$\begin{array}{l} z \\ \swarrow \\ au \\ z \end{array}$$

$$\left(\begin{array}{l} = \frac{7+26i}{-4+3i} \cdot \frac{4+3i}{4+3i} = \frac{28+21i+104i-78}{-16-9} \\ = \frac{-50+125i}{-25} = 2-5i \end{array} \right)$$

⚠ possible mais ce n'est pas le conjugué de $-4+3i$

$$= \frac{7+26i}{-4+3i} \cdot \frac{-4-3i}{-4-3i} = \frac{-28-21i-104i+78}{16+9}$$

$$= \frac{50-125i}{25} = 2-5i$$

$$\Rightarrow S = \{2-5i\}$$

Ex 1.1.4

$$\begin{cases} (2+i)z + (2-i)w = 7-4i \\ (1+i)z - iw = 2+i \end{cases} \begin{array}{l} \cdot i \\ \cdot (2-i) \end{array}$$

$$\Leftrightarrow \begin{cases} (2i-1)z + (2i+1)w = 7i+4 \\ (1+i)(2-i)z - (2i+1)w = 4+1 \end{cases} \Rightarrow \begin{array}{l} (-1+2i)z + (2i+1)w = 4+7i \\ (3+i)z - (2i+1)w = 5 \end{array}$$

$$\frac{(2+3i)z}{-} = 9+7i$$

$$\Rightarrow z = \frac{9+7i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{18-27i+14i+21}{4+9} = \frac{39-13i}{13} = 3-i$$

Ex 1.1.4

(suite)

$$\Rightarrow (1+i)(3-i) - iw = 2+i$$

$$-iw = 2+i - (3-i+3i+1) = -2-i$$

$$w = \frac{2+i}{1} \cdot \frac{i}{i} = \frac{-1+2i}{-1} = 1-2i$$

$$\Rightarrow \underline{S = \{1-2i; 3-i\}}$$

Ex 1.1.5

$$a) \underline{\bar{z}_1 = 5+4i} \quad b) \underline{-8+i} \quad c) \underline{2-3i} \quad d) \underline{5+\frac{3}{2}i}$$

Ex 1.1.6

$$\begin{aligned} & (7-8i)\overline{(8-7i)} - \overline{(4+3i)}(4-2i) \\ &= (7-8i)(8+7i) - (4-3i)(4-2i) \\ &= 56 + 49i - 64i + 56 - (16 - 8i - 12i - 6) = \underline{102+5i} \end{aligned}$$

Ex 1.1.7

On pose $z = x+yi$ Rappel: $a+bi = c+di \Leftrightarrow a=c$ et $b=d$

$$a) 8z + 5\bar{z} = 4+3i$$

$$\Leftrightarrow 8(x+yi) + 5(x-yi) = 4+3i$$

$$\Leftrightarrow 13x + 3yi = 4+3i$$

$$\Leftrightarrow \begin{cases} 13x = 4 \\ 3y = 3 \end{cases} \Leftrightarrow \begin{cases} x = \frac{4}{13} \\ y = 1 \end{cases} \Leftrightarrow z = \frac{4}{13} + i \Rightarrow \underline{S = \left\{ \frac{4}{13} + i \right\}}$$

$$b) z^2 + 2\bar{z} + 5 = 0$$

$$\Leftrightarrow (x+yi)^2 + 2(x-yi) + 5 = 0$$

$$\Leftrightarrow x^2 + 2xyi - y^2 + 2x - 2yi + 5 = 0$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 + 2x + 5 = 0 \\ -2xy - 2y = 0 \end{cases} \quad | :2 \quad \Leftrightarrow \begin{cases} x^2 - y^2 + 2x + 5 = 0 \\ y(x-1) = 0 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ y=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=1 \\ 1-y^2+2+5=0 \end{cases} \quad \text{ou} \quad \begin{cases} y=0 \\ x^2+2x+5=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=1 \\ y^2=8 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ y=\pm\sqrt{8} \end{cases}$$

$\Delta = 4 - 20 < 0$
impossible car $x \in \mathbb{R}$

$$\Rightarrow \underline{S = \{1+2\sqrt{2}i; 1-2\sqrt{2}i\}}$$

$$c) \quad 2 \overset{\text{Im}}{\Im}(\bar{z}+1) + 2i \overset{\text{Re}}{\Re}(-z+2) = -1-12i \quad (z=x+yi \quad \bar{z}=x-yi)$$

$$\Leftrightarrow 2 \Im(x+1-yi) + 2i \Re(-x+2-yi) = -1-12i$$

$$\Leftrightarrow 2 \cdot (-y) + 2i \cdot (-x+2) = -1-12i$$

$$\Leftrightarrow -2y - 2xi + 4i = -1-12i$$

$$\Leftrightarrow \begin{cases} -2y = -1 \\ -2x+4 = -12 \end{cases} \Leftrightarrow \begin{cases} y = \frac{1}{2} \\ x = 8 \end{cases} \Rightarrow \underline{S = \left\{ 8 + \frac{1}{2}i \right\}}$$

Ex 1.1.8

$$1) \quad z + \bar{z} = 2\Re(z) \quad \text{On pose } z = x+yi$$

$$z + \bar{z} = x+yi + x-yi = 2x = 2\Re(z) \quad \#$$

$$\Rightarrow \Re(z) = \frac{z + \bar{z}}{2}$$

$$2) \quad z - \bar{z} = 2\Im(z)i$$

$$z - \bar{z} = x+yi - x+yi = 2yi = 2\Im(z)i \quad \#$$

$$\Rightarrow \Im(z) = \frac{z - \bar{z}}{2}$$

$$3) \quad z\bar{z} = \Re(z)^2 + \Im(z)^2$$

$$(x+yi)(x-yi) = x^2 + y^2 = \Re(z)^2 + \Im(z)^2 \quad \#$$