Ex 2.6.2

a) $\lim_{x\to 1}(4x^3-2x^2+x-1)=4-2+1-1=2$

b) $\lim_{x\to -2}(x^2-5x)=4+10=14$

c) $\lim_{x\to 0}\frac{x+3x^2}{x+1}=\frac{0}{1}=0$

d) $\lim_{x\to 4}(-5)=-5$

e) $\lim_{x\to 3}\sqrt{x^2-5}=\sqrt{9-5}=2$

f) $\lim_{x\to 1}\frac{x^2+2x+1}{x^3+x^2+x}=\frac{1+2+1}{1+1+1}=\frac{4}{3}$

g) $\lim_{x\to \frac{\pi}{2}} \cos(x+\frac{\pi}{2}) = \cos(\pi) = -1$

h) $\lim_{x\to 0} \frac{\sin(x)+1}{2-\tan(x)} = \frac{0+1}{2-0} = \frac{1}{2}$

Ex 2.6.3

a) $\lim_{x\to 3}\frac{x-3}{2x-6}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim_{x\to 3}\frac{x-3}{2(x-3)}=\lim_{x\to 3}\frac{1}{2}=\frac{1}{2}$

b) $\lim_{x\to 0}\frac{100x^2}{x}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim_{x\to 0}\frac{100x}{1}=0$

c) $\lim_{x\to 2}\frac{(x-2)(x+1)}{(x+4)(x-2)}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim_{x\to 2}\frac{x+1}{x+4}=\frac{3}{6}=\frac{1}{2}$

d) $\lim_{x\to 1}\frac{x^2-1}{x-1}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}=\lim_{x\to 1}\frac{x+1}{1}=2$

e) $\lim_{x\to 5}\frac{x^2+2x-15}{x^2+8x+15}=\frac{20}{80}=\frac{1}{4}$

f) $\lim_{x\to 4}\frac{x^2-4x}{x^2-16}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim_{x\to 4}\frac{x(x-4)}{(x-4)(x+4)}=\lim_{x\to 4}\frac{x}{x+4}=\frac{4}{8}=\frac{1}{2}$

g) $\lim_{x\to 1}\frac{x^2-x}{x^3-4x^2-7x+10}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim_{x\to 1}\frac{x(x-1)}{x(x^2-3x-10)}=\lim_{x\to 1}\frac{x}{x^2-3x-10}=-\frac{1}{12}$

$$\begin{array}{c|cccc}
 & 1 & -4 & -7 & 10 \\
1 & & 1 & -3 & -10 \\
\hline
 & 1 & -3 & -10 & 0
\end{array}$$

h) $\lim_{x\to 1}\frac{2x^3-3x^2+1}{x^2+3x-4}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim_{x\to 1}\frac{(x-1)(2x^2-x-1)}{(x+4)(x-1)}=\lim_{x\to 1}\frac{2x^2-x-1}{x+4}=\frac{0}{5}=0$

$$\begin{array}{c|cccc}
 & 2 & -3 & 0 & 1 \\
1 & & 2 & -1 & -1 \\
\hline
 & 2 & -1 & -1 & 0
\end{array}$$

Ex 2.6.4

a) $\lim\limits_{x\to 16} \frac{x-16}{\sqrt{x}-4} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 16} \frac{x-16}{\sqrt{x}-4}\cdot\frac{\sqrt{x}+4}{\sqrt{x}+4} = \lim\limits_{x\to 16} \frac{\cancel{(x-16)}(\sqrt{x}+4)}{\cancel{x-16}} = \frac{\sqrt{16}+4}{1} = \underline{8}$

b) $\lim\limits_{x\to 25} \frac{\sqrt{x}-5}{x-25} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 25} \frac{\sqrt{x}-5}{x-25}\cdot\frac{\sqrt{x}+5}{\sqrt{x}+5} = \lim\limits_{x\to 25} \frac{\cancel{x-25}}{\cancel{(x-25)}(\sqrt{x}+5)} = \frac{1}{\sqrt{25}+5} = \underline{\frac{1}{10}}$

c) $\lim\limits_{h\to 0} \frac{4-\sqrt{16+h}}{h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{h\to 0} \frac{4-\sqrt{16+h}}{h}\cdot\frac{4+\sqrt{16+h}}{4+\sqrt{16+h}} = \lim\limits_{h\to 0} \frac{16-(16+h)}{h(4+\sqrt{16+h})}$

$= \lim\limits_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(4+\sqrt{16+h})} = \frac{-1}{4+\sqrt{16+0}} = \underline{-\frac{1}{8}}$

d) $\lim\limits_{x\to 0} \frac{x^2}{\sqrt{x^2+1}-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \frac{x^2}{\sqrt{x^2+1}-1}\cdot\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1} = \lim\limits_{x\to 0} \frac{\cancel{x^2}(\sqrt{x^2+1}+1)}{\underbrace{x^2+1-1}_{\cancel{x^2}\quad 1}} = \underline{2}$

e) $\lim\limits_{x\to 5} \frac{x-5}{\sqrt{2x-1}-3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 5} \frac{x-5}{\sqrt{2x-1}-3}\cdot\frac{\sqrt{2x-1}+3}{\sqrt{2x-1}+3} = \lim\limits_{x\to 5} \frac{(x-5)(\sqrt{2x-1}+3)}{2x-1-9}$

$= \lim\limits_{x\to 5} \frac{\cancel{(x-5)}(\sqrt{2x-1}+3)}{2\cancel{(x-5)}} = \frac{3+3}{2} = \underline{3}$

f) $\lim\limits_{t\to 2} \frac{1+\sqrt{t-2}}{t} = \underline{\frac{1}{2}}$

g) $\lim\limits_{x\to 2} \frac{x-2}{\sqrt{x+1}-\sqrt{2x-1}} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 2} \frac{x-2}{\sqrt{x+1}-\sqrt{2x-1}}\cdot\frac{\sqrt{x+1}+\sqrt{2x-1}}{\sqrt{x+1}+\sqrt{2x-1}}$

$= \lim\limits_{x\to 2} \frac{(x-2)(\sqrt{x+1}+\sqrt{2x-1})}{x+1-(2x-1)} = \lim\limits_{x\to 2} \frac{\cancel{(x-2)}(\sqrt{x+1}+\sqrt{2x-1})}{-\cancel{(x-2)}} = \frac{\sqrt{3}+\sqrt{3}}{-1} = \underline{-2\sqrt{3}}$

h) $\lim\limits_{x\to -2} \frac{x+\sqrt{x+6}}{x+\sqrt{2-x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to -2} \frac{x+\sqrt{x+6}}{x+\sqrt{2-x}}\cdot\frac{x-\sqrt{2-x}}{x-\sqrt{2-x}}\cdot\frac{x-\sqrt{x+6}}{x-\sqrt{x+6}} = \lim\limits_{x\to -2} \frac{(x^2-(x+6))(x-\sqrt{2-x})}{(x^2-(2-x))(x-\sqrt{x+6})}$

$= \lim\limits_{x\to -2} \frac{\cancel{(x+2)}(x-3)(x-\sqrt{2-x})}{\cancel{(x+2)}(x-1)(x-\sqrt{x+6})} = \frac{-5\cdot(-4)}{-3\cdot(-4)} = \underline{\frac{5}{3}}$

Ex 2.6.5

a) $f(x) = \dfrac{|x|}{x} = \begin{cases} \dfrac{-x}{x} & \text{si } x<0 \\ \dfrac{x}{x} & \text{si } x>0 \end{cases} = \begin{cases} -1 & \text{si } x<0 \\ 1 & \text{si } x>0 \end{cases}$ $\qquad ED(f) = \mathbb{R}^*$

$\lim\limits_{x \to 0_-} f(x) = -1 \neq \lim\limits_{x \to 0_+} f(x) = 1 \Rightarrow \lim\limits_{x \to 0} f(x)$ n'existe pas

b) $f(x) = \dfrac{x^2 + |x|}{|x|} = \begin{cases} \dfrac{x^2+x}{x} & \text{si } x>0 \\ \dfrac{x^2-x}{-x} & \text{si } x<0 \end{cases}$ $\qquad ED(f) = \mathbb{R}^*$

$\lim\limits_{\substack{x \to 0 \\ >}} f(x) \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\substack{x \to 0 \\ >}} \dfrac{x(x+1)}{x} = \lim\limits_{\substack{x \to 0 \\ >}} (x+1) = 1$

$\lim\limits_{\substack{x \to 0 \\ <}} f(x) \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\substack{x \to 0 \\ <}} \dfrac{x(x-1)}{-x} = \lim\limits_{\substack{x \to 0 \\ <}} -(x-1) = 1$

$\Rightarrow \lim\limits_{x \to 0} f(x) = 1$

c) $f(x) = \dfrac{x^2-2x}{|x|} = \begin{cases} \dfrac{x^2-2x}{x} & \text{si } x>0 \\ \dfrac{x^2-2x}{-x} & \text{si } x<0 \end{cases}$ $\qquad ED(f) = \mathbb{R}^*$

$\lim\limits_{\substack{x \to 0 \\ >}} \dfrac{x^2-2x}{x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\substack{x \to 0 \\ >}} \dfrac{x(x-2)}{x} = \lim\limits_{\substack{x \to 0 \\ >}} (x-2) = -2$

$\lim\limits_{\substack{x \to 0 \\ <}} \dfrac{x^2-2x}{-x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\substack{x \to 0 \\ <}} \dfrac{x(x-2)}{-x} = \lim\limits_{\substack{x \to 0 \\ <}} -(x-2) = 2$

$\Rightarrow \lim\limits_{x \to 0} f(x)$ n'existe pas

d) $f(x) = \dfrac{|x-2|}{x^2-3x+2} = \begin{cases} \dfrac{x-2}{x^2-3x+2} & \text{si } x>2 \\ \dfrac{-(x-2)}{x^2-3x+2} & \text{si } x<2 \end{cases}$ $\qquad ED(f) = \mathbb{R} - \{1;2\}$

$\lim\limits_{\substack{x \to 2 \\ >}} f(x) \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\substack{x \to 2 \\ >}} \dfrac{\cancel{x-2}}{\cancel{(x-2)}(x-1)} = \lim\limits_{\substack{x \to 2 \\ >}} \dfrac{1}{x-1} = 1$

$\lim\limits_{\substack{x \to 2 \\ <}} f(x) \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\substack{x \to 2 \\ <}} \dfrac{-\cancel{(x-2)}}{\cancel{(x-2)}(x-1)} = \lim\limits_{\substack{x \to 2 \\ <}} \dfrac{-1}{x-1} = -1$

$\Rightarrow \lim\limits_{x \to 2} f(x)$ n'existe pas

Ex 2.6.6

$$\lim_{x\to 0} x^2\sin\left(\frac{1}{x}\right) = 0 \quad \text{car :} \quad \begin{aligned} -1 &\leq \sin\left(\tfrac{1}{x}\right) \leq 1 \\ -x^2 &\leq x^2\sin\left(\tfrac{1}{x}\right) \leq x^2 \\ 0 &\leq \lim_{x\to 0} x^2\sin\left(\tfrac{1}{x}\right) \leq 0 \end{aligned} \quad \left| \begin{array}{l} \cdot x^2 \\ \lim_{x\to 0}(\ ) \end{array} \right.$$

$$\lim_{x\to 0^+} \sqrt{x}\cos\left(\frac{1}{x}\right) = 0 \quad \text{car :} \quad \begin{aligned} -1 &\leq \cos\left(\tfrac{1}{x}\right) \leq 1 \\ -\sqrt{x} &\leq \sqrt{x}\cos\left(\tfrac{1}{x}\right) \leq \sqrt{x} \\ 0 &\leq \lim_{x\to 0} \sqrt{x}\cos\left(\tfrac{1}{x}\right) \leq 0 \end{aligned} \quad \left| \begin{array}{l} \cdot \sqrt{x} \\ \lim_{x\to 0^+}(\ ) \end{array} \right.$$

Ex 2.6.7

$$\begin{aligned} 0 &\leq f(x) \leq 3 \\ 0 &\leq x f(x) \leq 3x \\ 0 &\leq \lim_{x\to 0} x f(x) \leq 0 \end{aligned} \quad \left| \begin{array}{l} \cdot x \\ \lim_{x\to 0}(\ ) \end{array} \right.$$

$$\Rightarrow \lim_{x\to 0} x f(x) = 0$$

Ex 2.6.8

a) $$\lim_{x\to 0} \frac{\sin(2x)}{x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2\sin(x)\cos(x)}{x} = \lim_{x\to 0} \frac{\sin(x)}{x} \cdot 2\cos(x) = 1\cdot 2 = 2$$

b) $$\lim_{x\to 0} \frac{\sin(3x)}{\sin(2x)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{\cancel{\sin(x)}(4\cos^2(x)-1)}{2\cancel{\sin(x)}\cos(x)} = \frac{3}{2}$$

<u>ou</u> $$= \lim_{x\to 0} \frac{\cancel{\sin(x)}(3-4\sin^2(x))}{2\cancel{\sin(x)}\cos(x)} = \frac{3}{2}$$

<u>ou</u> $$= \lim_{x\to 0} \frac{\sin(3x)}{3x} \cdot \frac{2x}{\sin(2x)} \cdot \frac{3x}{2x} = \lim_{x\to 0} \frac{\sin(3x)}{3x} \cdot \frac{1}{\frac{\sin(2x)}{2x}} \cdot \frac{3}{2} = 1 \cdot \frac{1}{1} \cdot \frac{3}{2} = \frac{3}{2}$$

c) $\lim\limits_{x\to 0} \dfrac{\sin\left(\frac{x}{4}\right)}{5x} \left(= \text{"}\tfrac{0}{0}\text{"}\right)$ chgmt variable : $\frac{x}{4} = t \Leftrightarrow x = 4t$

$\Rightarrow x \to 0,\ t \to 0$

$$= \lim_{t\to 0} \frac{\sin(t)}{20t} = \lim_{t\to 0} \frac{\sin(t)}{t} \cdot \frac{1}{20} = 1 \cdot \frac{1}{20} = \underline{\frac{1}{20}}$$

ou $= \lim\limits_{x\to 0} \dfrac{\sin\left(\frac{x}{4}\right)}{\frac{x}{4}} \cdot \dfrac{1}{5\cancel{x}} \cdot \dfrac{\cancel{x}}{4} = 1 \cdot \dfrac{1}{5} \cdot \dfrac{1}{4} = \dfrac{1}{20}$

d) $\lim\limits_{x\to 0} \dfrac{\tan(7x)}{\sin(3x)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \dfrac{\sin(7x)}{\cos(7x)} \cdot \dfrac{1}{\sin(3x)}$

$$= \lim_{x\to 0} \frac{\sin(7x)}{7x} \cdot \frac{7\cancel{x}}{\cos(7x)} \cdot \frac{1}{\frac{\sin(3x)}{3x}} \cdot \frac{1}{3\cancel{x}} = 1 \cdot \frac{7}{1} \cdot \frac{1}{1} \cdot \frac{1}{3} = \underline{\frac{7}{3}}$$

e) $\lim\limits_{x\to 1} \dfrac{\sin(x-1)}{x-1} \left(= \text{"}\tfrac{0}{0}\text{"}\right)$ chgmt variable : $t = x-1 \Rightarrow x \to 1,\ t \to 0$

$$= \lim_{t\to 0} \frac{\sin(t)}{t} = \underline{1}$$

f) $\lim\limits_{x\to 0} \dfrac{\tan(x)}{x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \dfrac{\frac{\sin(x)}{\cos(x)}}{x} = \lim\limits_{x\to 0} \dfrac{\sin(x)}{\cos(x)} \cdot \dfrac{1}{x} = \lim\limits_{x\to 0} \dfrac{\sin(x)}{x} \cdot \dfrac{1}{\cos(x)} = 1 \cdot \dfrac{1}{1} = \underline{1}$

g) $\lim\limits_{x\to 0} \dfrac{1-\cos(x)}{\sin^2(x)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \dfrac{1-\cos(x)}{1-\cos^2(x)} = \lim\limits_{x\to 0} \dfrac{\cancel{1-\cos(x)}}{\cancel{(1-\cos(x))}(1+\cos(x))} = \underline{\dfrac{1}{2}}$

h) $\lim\limits_{x\to \pi/2} \dfrac{\cos(x)}{x-\frac{\pi}{2}} \left(= \text{"}\tfrac{0}{0}\text{"}\right)$ chgmt variable : $x - \frac{\pi}{2} = t \Rightarrow x \to \frac{\pi}{2},\ t \to 0$

$$= \lim_{t\to 0} \frac{\cos\left(t+\frac{\pi}{2}\right)}{t} = \lim_{t\to 0} \frac{-\sin(t)}{t} = \underline{-1}$$

Ex 2.6.14

a) $\lim_{x\to-2} \frac{x^2+3x+6}{(x+2)^2} \overset{\text{"}\frac{4}{0_+}\text{"}}{=} = +\infty$

b) $\lim_{x\to-3} \frac{x^2+2x-15}{x^2+8x+15} \overset{\text{"}\frac{-12}{0}\text{"}}{=} \begin{cases} \xrightarrow{x<-3} -\frac{12}{0_-} = +\infty \\ \xrightarrow{x>-3} -\frac{12}{0_+} = -\infty \end{cases} = \infty$

c) $\lim_{x\to0} \frac{x^2-3x}{x^3} \underset{f.i.}{\overset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x\to0} \frac{x(x-3)}{x^3} = \lim_{x\to0} \frac{x-3}{x^2} \overset{\text{"}\frac{-3}{0_+}\text{"}}{=} -\infty$

d) $\lim_{x\underset{>}{\to}5} \frac{x-3}{5-x} \overset{\frac{2}{0_-}}{=} -\infty$

e) $\lim_{x\to1} (2x^2-5x+3)\frac{1}{x-1} \underset{f.i.}{\overset{\text{"}0\cdot\infty\text{"}}{=}} \lim_{x\to1} \frac{(2x-3)(x-1)}{x-1} = \lim_{x\to1} (2x-3) = -1$

$\Delta = 1 \Rightarrow x_{1,2} = \frac{5\pm1}{4} = \begin{cases} 3/2 \\ 1 \end{cases}$

f) $\lim_{x\to1} \left(\frac{x^2}{x-1} - \frac{1}{x-1}\right) \underset{f.i.}{\overset{\text{"}\infty-\infty\text{"}}{=}} \lim_{x\to1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to1} (x+1) = 2$

g) $\lim_{x\underset{<}{\to}-2} \frac{x-1}{x+2} \overset{\text{"}\frac{-3}{0_-}\text{"}}{=} +\infty$

h) $\lim_{x\to2} \left(\frac{1}{x-2} - \frac{4}{x^2-4}\right) \underset{f.i.}{\overset{\text{"}\infty-\infty\text{"}}{=}} \lim_{x\to2} \frac{x+2-4}{(x+2)(x-2)} = \lim_{x\to2} \frac{x-2}{(x+2)(x-2)}$

$= \lim_{x\to2} \frac{1}{x+2} = \frac{1}{4}$

Ex 2.6.16

a) $\lim\limits_{x\to\infty} \frac{2x-4}{-3x+1} = \lim\limits_{x\to\infty} \frac{2x}{-3x} = -\frac{2}{3}$

b) $\lim\limits_{x\to-\infty} \frac{-3x^2+1}{x+2} = \lim\limits_{x\to-\infty} \frac{-3x^2}{x} = \lim\limits_{x\to-\infty} -3x = -3\cdot(-\infty) = +\infty$

c) $\lim\limits_{x\to\infty} \frac{x^2+4x+49}{x^2-2x+4} = \lim\limits_{x\to\infty} \frac{x^2}{x^2} = 1$

d) $\lim\limits_{x\to\infty} \frac{(3x+4)(x-1)}{(2x+7)(1-5x)} = \lim\limits_{x\to\infty} \frac{3x^2+\dots}{-10x^2+\dots} = \lim\limits_{x\to\infty} \frac{3x^2}{-10x^2} = -\frac{3}{10}$

e) $\lim\limits_{x\to\infty} \frac{(x+1)^7(2x+3)^4}{(2x+1)^3(x-98)^8} = \lim\limits_{x\to\infty} \frac{(x^7+\dots)(16x^4+\dots)}{(8x^3+12x^2+\dots)(x^8+\dots)} = \lim\limits_{x\to\infty} \frac{16x^{11}}{8x^{11}} = 2$

f) $\lim\limits_{x\to\infty} \left(\frac{2x^2-1}{x-1} + 1 - 2x\right) = \lim\limits_{x\to\infty} \frac{2x^2-1}{x-1} + \frac{(1-2x)(x-1)}{x-1} = \lim\limits_{x\to\infty} \frac{2x^2-1-2x^2+3x-1}{x-1}$

$= \lim\limits_{x\to\infty} \frac{3x-2}{x-1} = \lim\limits_{x\to\infty} \frac{3x}{x} = 3$

g) $\lim\limits_{x\to+\infty} \left(\frac{2x-x^3}{3x+1} + x - 1\right) = \lim\limits_{x\to+\infty} \left(\frac{2x-x^3}{3x+1} + \frac{(x-1)(3x+1)}{3x+1}\right) = \lim\limits_{x\to+\infty} \frac{2x-x^3+3x^2-2x-1}{3x+1}$

$= \lim\limits_{x\to+\infty} \frac{-x^3}{3x} = \lim\limits_{x\to+\infty} \frac{-x^2}{3} = \frac{-(+\infty)^2}{3} = -\infty$

h) $\lim\limits_{x\to\infty} \left(\frac{1+5x-3x^2}{x-2} + 3x + 1\right) = \lim\limits_{x\to\infty} \left(\frac{1+5x-3x^2}{x-2} + \frac{(3x+1)(x-2)}{x-2}\right)$

$= \lim\limits_{x\to\infty} \frac{1+5x-3x^2+3x^2-5x-2}{x-2} = \lim\limits_{x\to\infty} \frac{-1}{x-2} = \text{"}\frac{-1}{\infty}\text{"} = 0$

Ex 2.6.17

a) $\lim_{x\to-\infty} \sqrt{x^2+x+1}-x = +\infty-(-\infty) = +\infty+\infty = +\infty$

$\lim_{x\to+\infty} \sqrt{x^2+x+1}-x = \text{"}+\infty-(+\infty)\text{"} = \lim_{x\to+\infty} \frac{(\sqrt{x^2+x+1}-x)(\sqrt{x^2+x+1}+x)}{\sqrt{x^2+x+1}+x}$

f.i.

$= \lim_{x\to+\infty} \frac{\cancel{x^2}+x+1-\cancel{x^2}}{|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+x} = \lim_{x\to+\infty} \frac{\cancel{x}(1+\frac{1}{x})}{\cancel{x}(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1)} = \frac{1}{1+1} = \frac{1}{2}$

$|x| \to x$

b) $\lim_{x\to\pm\infty} \frac{\sqrt{4x^2-4x+3}}{x+1} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\pm\infty} \frac{|2x|\sqrt{1-\frac{1}{x}+\frac{3}{4x^2}}}{x(1+\frac{1}{x})} = \begin{cases} \xrightarrow{x\to-\infty} \frac{-2}{1} = -2 \\ \xrightarrow{x\to+\infty} \frac{2}{1} = 2 \end{cases}$

f.i.

c) $\lim_{x\to\pm\infty} \sqrt{x^2+2x}-\sqrt{x^2+4} = \text{"}+\infty-(+\infty)\text{"} = \lim_{x\to\pm\infty} \frac{(\sqrt{x^2+2x}-\sqrt{x^2+4})(\sqrt{x^2+2x}+\sqrt{x^2+4})}{\sqrt{x^2+2x}+\sqrt{x^2+4}}$

f.i.

$= \lim_{x\to\pm\infty} \frac{x^2+2x-(x^2-4)}{|x|\sqrt{1+\frac{2}{x}}+|x|\sqrt{1+\frac{4}{x}}} = \lim_{x\to\pm\infty} \frac{\cancel{x}(2+\frac{4}{x})}{|x|(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{4}{x}})}$

$|x| \to \pm\cancel{x}$

$= \begin{cases} \xrightarrow{x\to-\infty} \frac{2}{-(1+1)} = -1 \\ \xrightarrow{x\to+\infty} \frac{2}{1+1} = 1 \end{cases}$

d) $\lim_{x\to-\infty} \frac{2x-\sqrt{4x^2+2x-5}}{x+3} = \frac{\text{"}-\infty-(+\infty)\text{"}}{-\infty} = \frac{\text{"}-\infty\text{"}}{-\infty} = \lim_{x\to-\infty} \frac{2x-|2x|\sqrt{1+\frac{1}{2x}-\frac{5}{4x^2}}}{x(1+\frac{3}{x})}$

f.i. $\qquad |2x| \to -2x$

$= \lim_{x\to-\infty} \frac{2\cancel{x}(1+\sqrt{1+\frac{1}{2x}-\frac{5}{4x^2}})}{\cancel{x}(1+\frac{3}{x})} = \frac{2(1+1)}{1} = 4$

$$\lim_{x\to+\infty} \frac{2x-\sqrt{4x^2+2x-5}}{x+3} = \frac{\text{“}\overset{\text{f.i.}}{\infty-\infty}\text{”}}{+\infty} = \lim_{x\to+\infty} \frac{2x-\sqrt{4x^2+2x-5}}{x+3} \cdot \frac{2x+\sqrt{4x^2+2x-5}}{2x+\sqrt{4x^2+2x-5}}$$

$$= \lim_{x\to+\infty} \frac{4x^2-(4x^2+2x-5)}{(x+3)(2x+\sqrt{4x^2+2x-5})}$$

$$= \lim_{x\to+\infty} \frac{-2x+5}{(x+3)\left(2x+\underbrace{|2x|}_{2x}\sqrt{1+\frac{1}{2x}-\frac{5}{4x^2}}\right)}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}(-2+5/x)}{2x^{\cancel{2}}(1+\frac{3}{x})\left(1+\sqrt{1+\frac{1}{2x}-\frac{5}{4x^2}}\right)} = \frac{\text{“}-2\text{”}}{+\infty} = \underline{0}$$

e)

$$-1 \leq \cos(x) \leq 1 \qquad \Big| \cdot(-1)$$

$$1 \geq -\cos(x) \geq -1 \qquad \Big| +2x$$

$$2x+1 \geq 2x-\cos(x) \geq 2x-1 \qquad \Big| \lim_{x\to\pm\infty}(\;)$$

$$\pm\infty \geq \lim_{x\to\pm\infty}(2x-\cos(x)) \geq \pm\infty$$

$$\overset{\text{thm des}}{\underset{\text{2 gendarmes}}{\Longrightarrow}} \quad \lim_{x\to\pm\infty}(2x-\cos(x)) = \pm\infty$$

f)

$$-1 \leq \cos(x) \leq 1 \qquad \Big| \cdot(-1)$$

$$1 \geq -\cos(x) \geq -1 \qquad \Big| +2x$$

$$2x+1 \geq 2x-\cos(x) \geq 2x-1 \qquad \Big| \cdot\left(\frac{1}{x-1}\right)$$

$$\frac{2x+1}{x-1} \geq \frac{2x-\cos(x)}{x-1} \geq \frac{2x-1}{x-1} \qquad \Big| \lim_{x\to\pm\infty}(\;)$$

$$\lim_{x\to\pm\infty} \frac{2\cancel{x}}{\cancel{x}} \geq \lim_{x\to\pm\infty} \frac{2x-\cos(x)}{x-1} \geq \lim_{x\to\pm\infty} \frac{2\cancel{x}}{\cancel{x}}$$

$$2 \geq \lim_{x\to\pm\infty} \frac{2x-\cos(x)}{x-1} \geq 2$$

$$\overset{\text{thm des}}{\underset{\text{2 gendarmes}}{\Longrightarrow}} \quad \lim_{x\to\pm\infty} \frac{2x-\cos(x)}{x-1} = \underline{2}$$

g) $\lim_{x \to \pm\infty} x \sin\left(\frac{1}{x}\right) = \lim_{t \to 0} \frac{\sin(t)}{t} = \underline{1}$

chgmt variable $t = \frac{1}{x}$ et $x \to \infty$

$\frac{1}{x} \to 0$

h) $\lim_{x \to \pm\infty} \sin\left(\frac{1}{x}\right) = \lim_{t \to 0} \sin(t) = \sin(0) = \underline{0}$