Ex 1.4.1 (dans le plan)

a) $\left\|\begin{pmatrix}0\\-5\end{pmatrix}\right\| = \sqrt{0^2+(-5)^2} = \sqrt{0+25} = \underline{5}$

$\left\|\begin{pmatrix}3\\8\end{pmatrix}\right\| = \sqrt{3^2+8^2} = \sqrt{9+64} = \underline{\sqrt{73}}$

$\left\|\begin{pmatrix}1/2\\5/2\end{pmatrix}\right\| = \sqrt{\frac{1}{4}+\frac{25}{4}} = \sqrt{\frac{26}{4}} = \frac{\sqrt{26}}{\sqrt{4}} = \underline{\frac{\sqrt{26}}{2}}$ ou $\sqrt{\frac{26}{4}} = \underline{\sqrt{\frac{13}{2}}}$

$\left\|\begin{pmatrix}6/10\\-4/5\end{pmatrix}\right\| = \left\|\begin{pmatrix}3/5\\-4/5\end{pmatrix}\right\| = \sqrt{\frac{9}{25}+\frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = \underline{1}$

ou $\sqrt{\frac{36}{100}+\frac{16}{25}} = \sqrt{\frac{9}{25}+\frac{16}{25}} = \dots$

b) $\begin{pmatrix}-1/\sqrt{5}\\6/\sqrt{45}\end{pmatrix}$ est unitaire car $\sqrt{\frac{1}{5}+\frac{36}{45}} = \sqrt{\frac{1}{5}+\frac{4}{5}} = \sqrt{\frac{5}{5}} = \sqrt{1} = 1$

#

c) modifié : donner un vecteur unitaire au vecteurs suivants.

$\vec{a} = \begin{pmatrix}3\\4\end{pmatrix}$ $\quad \|\vec{a}\| = \sqrt{9+16} = 5 \Rightarrow \vec{u}_a = \frac{1}{5}\begin{pmatrix}3\\4\end{pmatrix} = \underline{\begin{pmatrix}3/5\\4/5\end{pmatrix} \text{ ou } \begin{pmatrix}-3/5\\-4/5\end{pmatrix}}$

$\vec{b} = \begin{pmatrix}12\\-5\end{pmatrix}$ $\quad \|\vec{b}\| = \sqrt{144+25} = 13 \Rightarrow \underline{\vec{u}_b = \frac{1}{13}\begin{pmatrix}12\\-5\end{pmatrix} = \begin{pmatrix}12/13\\-5/13\end{pmatrix} \text{ ou } \begin{pmatrix}-12/13\\5/13\end{pmatrix}}$

$\vec{c} = \begin{pmatrix}-6\\0\end{pmatrix}$ $\quad \|\vec{c}\| = \sqrt{36+0} = 6 \Rightarrow \vec{u}_c = \frac{1}{6}\begin{pmatrix}-6\\0\end{pmatrix} = \underline{\begin{pmatrix}-1\\0\end{pmatrix} \text{ ou } \begin{pmatrix}1\\0\end{pmatrix}}$

d) $\|\vec{a}\| = 10 \iff \left\|\begin{pmatrix}8\\k-1\end{pmatrix}\right\| = 10$

$\iff \sqrt{8^2+(k-1)^2} = 10 \qquad |\ ()^2$ et CL

$\iff 64+k^2-2k+1 = 100$

$\iff k^2-2k-35 = 0$

$\iff (k+5)(k-7) = 0 \qquad$ ou avec $\Delta = \dots$

$\Rightarrow k = \underline{-5}$: vérif : $\sqrt{8^2+(-6)^2} = \sqrt{64+36} = 10$

$\ \underline{7}$: " $\sqrt{8^2+6^2} = 10$

e) $\|\vec{u}+m\vec{v}\| = \sqrt{82} \iff \left\|\begin{pmatrix}2\\3\end{pmatrix}+m\begin{pmatrix}-2\\4\end{pmatrix}\right\| = \sqrt{82}$

$\iff \left\|\begin{pmatrix}2-2m\\3+4m\end{pmatrix}\right\| = \sqrt{82}$

$\iff \sqrt{(2-2m)^2+(3+4m)^2} = \sqrt{82} \quad | \ ()^2 + cl$

$\iff 4-8m+4m^2+9+24m+16m^2 = 82$

$\iff 20m^2+16m-69 = 0 \qquad \Delta = 5776 = 76^2$

$\Rightarrow m = \dfrac{-16\pm 76}{40} = \begin{cases} -\dfrac{23}{10} \\ \dfrac{3}{2} \end{cases}$

Ex 1.4.2 modifié

$A(2;1) \qquad B(4;3) \qquad C(2;6)$

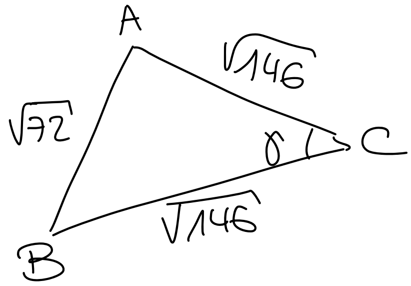
$\vec{AB} = \vec{OB}-\vec{OA} = \begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix} \Rightarrow \|\vec{AB}\| = \sqrt{4+4} = \sqrt{8}$

$\vec{AC} = \begin{pmatrix}0\\5\end{pmatrix} \Rightarrow \|\vec{AC}\| = \sqrt{0+25} = 5$

$\vec{BC} = \begin{pmatrix}-2\\3\end{pmatrix} \Rightarrow \|\vec{BC}\| = \sqrt{4+9} = \sqrt{13}$

$\Rightarrow$ périmètre $= \sqrt{8}+5+\sqrt{13}\ u \simeq 11,43\ u$

Ex 1.4.3



$A(6;4)$ $B(12;-2)$ $C(17;9)$

$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$

- $\|\overrightarrow{AC}\| = \|\overrightarrow{BC}\| = \sqrt{121+25} = \sqrt{146}$ $\Rightarrow$ le triangle est isocèle en C #

- $\mathcal{A} = \frac{1}{2}\left| \det(\overrightarrow{AB};\overrightarrow{AC}) \right| = \frac{1}{2}\left| \begin{vmatrix} 6 & 11 \\ -6 & 5 \end{vmatrix} \right| = \frac{1}{2}\left|6\cdot 5-(-6)\cdot 11\right| = \frac{1}{2}\cdot 96 = \underline{48\,u^2}$

Variante :

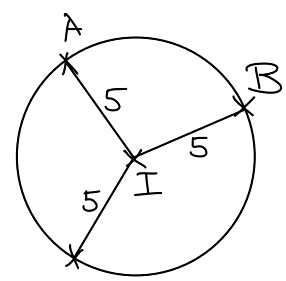
$\|\overrightarrow{AB}\| = \sqrt{36+36} = \sqrt{36\cdot 2} = 6\sqrt{2}$ $\Rightarrow$ $\frac{1}{2}\|\overrightarrow{AB}\| = \sqrt{18} = 3\sqrt{2}$

hauteur du Δ issue de C : $h = \sqrt{146-18} = \sqrt{128} = 8\sqrt{2}$

$\Rightarrow$ $\mathcal{A}_\Delta = \frac{1}{2}\cdot 6\sqrt{2}\cdot 8\sqrt{2} = 24\cdot 2 = \underline{48\ u^2}$ (car Δ isocèle)

Ex 1.4.4

$A(7;1)$ $B(5;5)$ $C(5;-3)$ $I(2;1)$



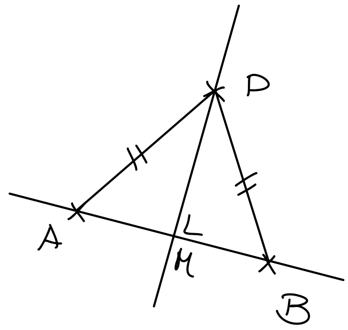
$\overrightarrow{AI} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$ $\overrightarrow{BI} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ $\overrightarrow{CI} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$\|\overrightarrow{AI}\| = \|\overrightarrow{BI}\| = \|\overrightarrow{CI}\| = 5$

$\Rightarrow$ A, B, C sont sur le cercle de centre I et de rayon 5 u #

Ex 1.4.5

$P(2;-1) \quad A(5;3) \quad B(-2;k)$



$\vec{PA} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad \vec{PB} = \begin{pmatrix} -4 \\ k+1 \end{pmatrix}$

P sur la médiatrice $\Leftrightarrow \|\vec{PA}\| = \|\vec{PB}\|$

$\Leftrightarrow 5 = \sqrt{16 + (k+1)^2} \quad | \ ()^2$

$\Leftrightarrow 25 = 16 + k^2 + 2k + 1$

$\Leftrightarrow k^2 + 2k - 8 = 0$

$\Leftrightarrow (k+4)(k-2) = 0$

$\Rightarrow k = -4$ ou 2

vérif : $5 = \sqrt{16 + (-3)^2}$ ✓

$5 = \sqrt{16 + 3^2}$ ✓

Ex 1.4.6 modifié

$A(1;2) \quad B(3;8) \quad P(2;y)$

$\vec{PA} = \begin{pmatrix} 1-2 \\ 2-y \end{pmatrix} = \begin{pmatrix} -1 \\ 2-y \end{pmatrix} \qquad \vec{PB} = \begin{pmatrix} 3-2 \\ 8-y \end{pmatrix} = \begin{pmatrix} 1 \\ 8-y \end{pmatrix}$

P sur la médiatrice $\Leftrightarrow \|\vec{PA}\| = \|\vec{PB}\|$

$\Leftrightarrow \sqrt{(-1)^2 + (2-y)^2} = \sqrt{1^2 + (8-y)^2} \quad | \ ()^2$

$\Leftrightarrow 1 + 4 - 4y + \cancel{y^2} = 1 + 64 - 16y + \cancel{y^2}$

$\Leftrightarrow 12y = 60$

$\Leftrightarrow y = 5$

vérif : $\sqrt{(-1)^2 + (-3)^2} = \sqrt{1^2 + 3^2}$ ✓

Ex 1.4.9

a) $\vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 3 \end{pmatrix} = 2 \cdot 7 + (-5) \cdot 3 = 14 - 15 = -1 \neq 0 \iff \vec{a} \not\perp \vec{b}$

b) $\begin{pmatrix} 53 \\ -41 \end{pmatrix} \cdot \begin{pmatrix} 41 \\ 53 \end{pmatrix} = 53 \cdot 41 + (-41) \cdot 53 = 0 \iff \vec{a} \perp \vec{b}$

c) $\begin{pmatrix} -8 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 16 \end{pmatrix} = -8 \cdot 6 + 3 \cdot 16 = -48 + 48 = 0 \iff \vec{a} \perp \vec{b}$

d) $\begin{pmatrix} 1/2 \\ -1/3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 3/4 \end{pmatrix} = \frac{1}{2} \cdot 5 + \left(-\frac{1}{3}\right) \cdot \frac{3}{4} = \frac{5}{2} - \frac{1}{4} = \frac{10-1}{4} = \frac{9}{4} \neq 0 \iff \vec{a} \not\perp \vec{b}$

Ex 1.4.11

a) $\begin{pmatrix} m \\ -2 \end{pmatrix} \perp \begin{pmatrix} 3 \\ 5 \end{pmatrix} \iff \begin{pmatrix} m \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 5 \end{pmatrix} = 0 \iff 3m - 10 = 0 \iff m = \frac{10}{3}$

c) $\vec{v} = k\vec{u} + \vec{w} \iff \vec{w} = \vec{v} - k\vec{u} = \begin{pmatrix} -3 \\ 11 \end{pmatrix} - k \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -3-k \\ 11-3k \end{pmatrix}$

$\vec{u} \perp \vec{w} \iff \vec{u} \cdot \vec{w} = 0$

$\iff \begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3-k \\ 11-3k \end{pmatrix} = 0$

$\iff 1 \cdot (-3-k) + 3(11 - 3k) = 0$

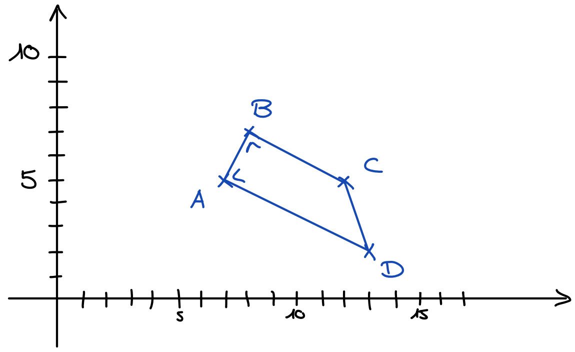
$\iff -3 - k + 33 - 9k = 0$

$\iff -10k + 30 = 0 \iff k = 3$

$\Rightarrow \vec{w} = \begin{pmatrix} -3-3 \\ 11 - 3 \cdot 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

Ex 1.4.12

$A(7;5)$ $B(8;7)$ $C(12;5)$ $D(13;2)$



- Pour prouver que ABCD est un trapèze rectangle en A et B (cf représentation) il suffit de prouver que les angles en A et en B sont droits.

$\vec{AB} = \begin{pmatrix}8\\7\end{pmatrix} - \begin{pmatrix}7\\5\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$ $\vec{AD} = \begin{pmatrix}13\\2\end{pmatrix} - \begin{pmatrix}7\\5\end{pmatrix} = \begin{pmatrix}6\\-3\end{pmatrix}$ $\vec{BC} = \begin{pmatrix}12\\5\end{pmatrix} - \begin{pmatrix}8\\7\end{pmatrix} = \begin{pmatrix}4\\-2\end{pmatrix}$

$$\left.\begin{array}{l} \vec{AB}\cdot\vec{AD} = 6-6 = 0 \iff \vec{AB} \perp \vec{AD} \\ \vec{AB}\cdot\vec{BC} = 4-4 = 0 \iff \vec{AB} \perp \vec{BC} \end{array}\right\} \Rightarrow$$ ABCD est un trap. rectangle en A et B

#

- $\|\vec{AB}\| = \sqrt{1+4} = \sqrt{5}$

$\|\vec{AD}\| = \sqrt{36+9} = \sqrt{45} = \sqrt{9\cdot 5} = 3\sqrt{5}$

$\|\vec{BC}\| = \sqrt{16+4} = \sqrt{20} = \sqrt{4\cdot 5} = 2\sqrt{5}$

$$\Rightarrow \mathcal{A} = \frac{3\sqrt{5}+2\sqrt{5}}{2}\cdot\sqrt{5} = \frac{5\sqrt{5}}{2}\cdot\sqrt{5} = \frac{5\cdot 5}{2} = \frac{25}{2}\ u^2$$

Ex 1.4.14

a) $\underbrace{\vec{a}\cdot(\underbrace{7\vec{b}+\vec{c}}_{\text{vecteur}})}_{\text{nbre}} = \begin{pmatrix}3\\4\end{pmatrix}\cdot\begin{pmatrix}35+7\\-7+1\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}\cdot\begin{pmatrix}42\\-6\end{pmatrix} = 3\cdot 42+4\cdot(-6) = 126-24 = \underline{102}$

b) $\underbrace{(\underbrace{\vec{a}\cdot\vec{b}}_{\text{nbre}})\,\vec{b}}_{\text{vecteur}} = (3\cdot 5+4\cdot(-1))\begin{pmatrix}5\\-1\end{pmatrix} = 11\begin{pmatrix}5\\-1\end{pmatrix} = \underline{\begin{pmatrix}55\\-11\end{pmatrix}}$

c) $\underbrace{(\underbrace{\vec{a}\cdot\vec{b}}_{\text{nbre}})+(\underbrace{\vec{c}\cdot\vec{d}}_{\text{nbre}})}_{\text{nbre}} = 11+(7\cdot 0+1\cdot 3) = 11+3 = \underline{14}$

d) $\underbrace{(\underbrace{\vec{a}+\vec{b}}_{\text{vect.}})\cdot(\underbrace{\vec{c}-\vec{d}}_{\text{vect.}})}_{\text{nbre}} = \begin{pmatrix}8\\3\end{pmatrix}\cdot\begin{pmatrix}7\\-2\end{pmatrix} = 8\cdot 7+3\cdot(-2) = 56-6 = \underline{50}$

e) $\underbrace{\underbrace{\|\vec{d}\|}_{\text{nbre}}\,(\underbrace{\vec{a}\cdot\vec{d}}_{\text{nbre}})}_{\text{nbre}} = \sqrt{0+9}\,(3\cdot 0+4\cdot 3) = 3\cdot 12 = \underline{36}$

f) $\underbrace{\underbrace{\vec{a}}_{\text{vect.}} + (\underbrace{\vec{b}\cdot\vec{c}}_{\text{nbre}})}_{\text{pas défini}}$

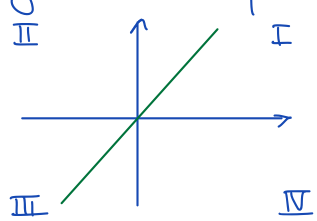
vect. + nbre

pas défini on ne peut pas additionner un vecteur et un nbre !

Ex 1.4.16

$A(-2;4)$ $B(1;-2)$ et $C(\lambda;\lambda)$

(C'est sur la diagonale du quadrant I ou III)



a) ΔABC rectangle en A

$\vec{AB} = \begin{pmatrix}1+2\\-2-4\end{pmatrix} = \begin{pmatrix}3\\-6\end{pmatrix}$

$\vec{AC} = \begin{pmatrix}\lambda+2\\\lambda-4\end{pmatrix}$

$\vec{AB}\perp\vec{AC} \Leftrightarrow \vec{AB}\cdot\vec{AC} = 0 \Leftrightarrow 3(\lambda+2)+(-6)(\lambda-4) = 0$

$\Leftrightarrow 3\lambda+6-6\lambda+24 = 0$

$\Leftrightarrow -3\lambda+30 = 0$

$\Leftrightarrow \lambda = 10 \quad\Rightarrow\quad \underline{C(10;10)}$

b) ΔABC rectangle en B

$$\vec{AB} = \begin{pmatrix} 1+2 \\ -2-4 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix} \lambda - 1 \\ \lambda + 2 \end{pmatrix}$$

$$\vec{AB} \perp \vec{BC} \iff \vec{AB} \cdot \vec{BC} = 0 \iff 3(\lambda - 1) + (-6)(\lambda+2) = 0$$

$$\iff 3\lambda - 3 - 6\lambda - 12 = 0$$

$$\iff -3\lambda - 15 = 0$$

$$\iff \lambda = -5 \implies \underline{C_2(-5\,;-5)}$$

c) ΔABC rectangle en C

$$\vec{AC} = \begin{pmatrix} \lambda + 2 \\ \lambda - 4 \end{pmatrix}$$

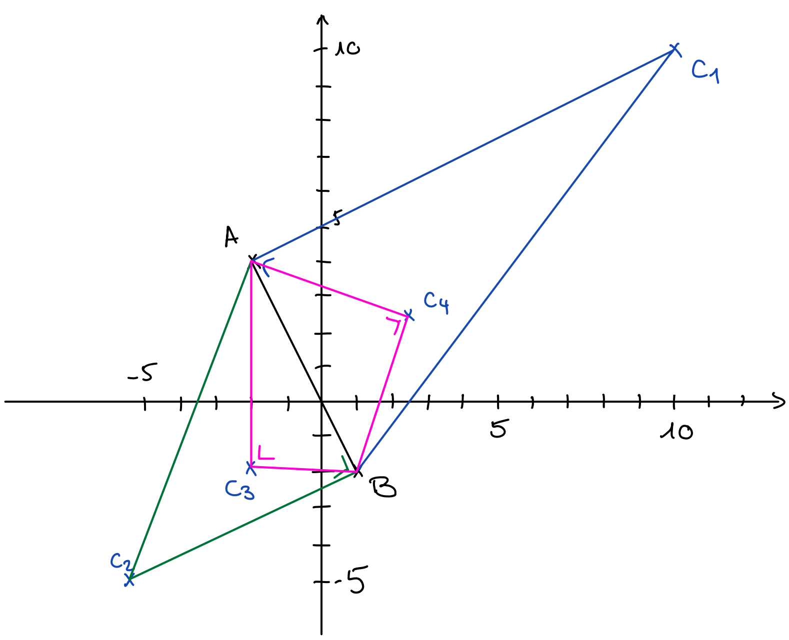
$$\vec{BC} = \begin{pmatrix} \lambda - 1 \\ \lambda + 2 \end{pmatrix}$$

$$\vec{AC} \perp \vec{BC} \iff \vec{AC} \cdot \vec{BC} = 0 \iff (\lambda+2)(\lambda-1) + (\lambda-4)(\lambda+2) = 0$$

$$\iff \lambda^2 + 2\lambda - \lambda - 2 + \lambda^2 - 4\lambda + 2\lambda - 8 = 0$$

$$\iff 2\lambda^2 - \lambda - 10 = 0 \qquad \Delta = 81$$

$$\implies \lambda_{1,2} = \frac{1 \pm 9}{4} = \begin{cases} -2 & \implies \underline{C_3(-2\,;-2)} \\ \frac{5}{2} & \implies \underline{C_4\left(\frac{5}{2}\,;\frac{5}{2}\right)} \end{cases}$$



C_3 et C_4 sont sur le cercle de Thalès de AB.

1.4.17

rectangle en A : $4(k-1)-2=0 \Leftrightarrow 4k-6=0 \Leftrightarrow k=3/2 \Rightarrow C\left(\frac{3}{2};5\right)$

rectangle en B : $4(k-5)-6=0 \Leftrightarrow 4k-26=0 \Leftrightarrow k=13/2 \Rightarrow C'\left(\frac{13}{2};5\right)$

rectangle en C : $\vec{CB}=\begin{pmatrix}5-k\\-3\end{pmatrix}$ $\quad \vec{CA}=\begin{pmatrix}1-k\\-1\end{pmatrix}$

$$\vec{CB}\perp\vec{CA} \Leftrightarrow \vec{CB}\cdot\vec{CA}=0$$

$$\Leftrightarrow \begin{pmatrix}5-k\\-3\end{pmatrix}\begin{pmatrix}1-k\\-1\end{pmatrix}=0$$

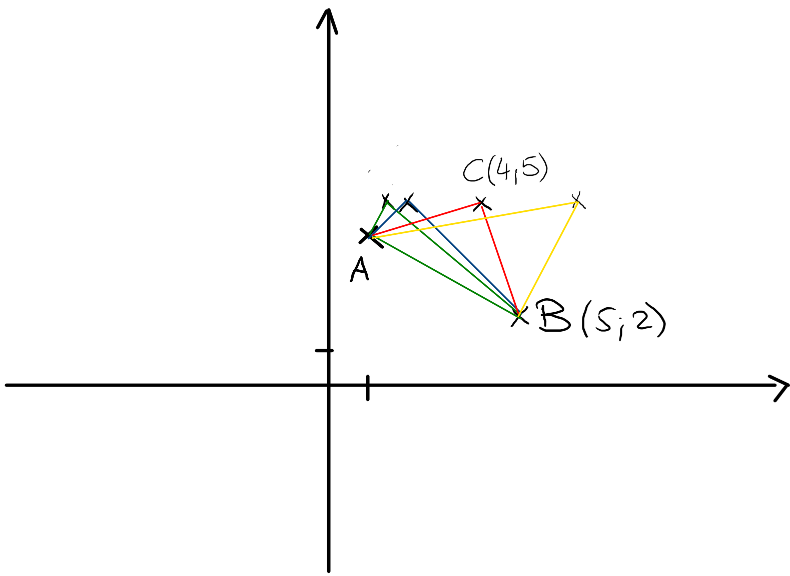
$$= (5-k)(1-k)+3=0$$

$$\Leftrightarrow 5-5k-k+k^2+3=0$$

$$\Leftrightarrow k^2-6k+8=0$$

$$\Leftrightarrow (k-4)(k-2)=0$$

$$k=4 \qquad k=2$$



$$\Rightarrow C_1(2;5) \quad \text{ou} \quad C_2(4;5)$$

$$\|\vec{AC_2}\| = \left\|\begin{pmatrix}4-1\\5-4\end{pmatrix}\right\| = \left\|\begin{pmatrix}3\\1\end{pmatrix}\right\| = \sqrt{9+1}=\sqrt{10}$$

$$\|\vec{BC_2}\| = \left\|\begin{pmatrix}4-5\\5-2\end{pmatrix}\right\| = \left\|\begin{pmatrix}-1\\3\end{pmatrix}\right\| = \sqrt{1+9}=\sqrt{10}$$

$= \checkmark$

Ex 1.4.28

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \qquad \overrightarrow{CD} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \qquad \Rightarrow \left\{ \begin{array}{l} \overrightarrow{AB} \cdot \overrightarrow{CD} = -30+0 = -30 \\ \|\overrightarrow{AB}\| = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10} \\ \|\overrightarrow{CD}\| = 5 \end{array} \right.$$

$$\cos(\alpha) = \frac{-30}{2\sqrt{10} \cdot 5} = \frac{-3}{\sqrt{10}} \iff \alpha = 161{,}57°$$

$\Rightarrow$ l'angle aigu : $180° - \alpha \cong 18{,}43°$

Ex 1.4.30

$$\left(\frac{3+1+x}{3} ; \frac{1-3+y}{3} \right) \Rightarrow y = 2$$

$$G(x_G; 0) \quad \Rightarrow \quad C(x; 2)$$

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix} x-3 \\ 1 \end{pmatrix}$$

$$\text{aire du } \Delta\, ABC = \frac{|(-2) \cdot 1 - (-4) \cdot (x-3)|}{2} = 3 \quad \Rightarrow \quad |4x - 14| = 6$$

$$\Rightarrow \quad 4x - 14 = \pm 6 \quad \Rightarrow \quad x_1 = 5 \qquad x_2 = 2$$

$\Rightarrow \quad C_1(5;2)$ ou $C_2(2;2)$

Ex 1.4.31

$$\overrightarrow{AB} = \overrightarrow{DC} = \begin{pmatrix} 9 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{DC} = \begin{pmatrix} -8 \\ 4 \end{pmatrix} \quad \Rightarrow \quad D(-8; 4)$$

$$\overrightarrow{AD} = \begin{pmatrix} -6 \\ 5 \end{pmatrix} \quad \Rightarrow \quad \det(\overrightarrow{AB}; \overrightarrow{AD}) = \begin{vmatrix} 9 & -6 \\ 1 & 5 \end{vmatrix} = 9 \cdot 5 - 1 \cdot (-6) = 51$$

$$\Rightarrow \quad \mathcal{A} = |51| = 51\ u^2$$

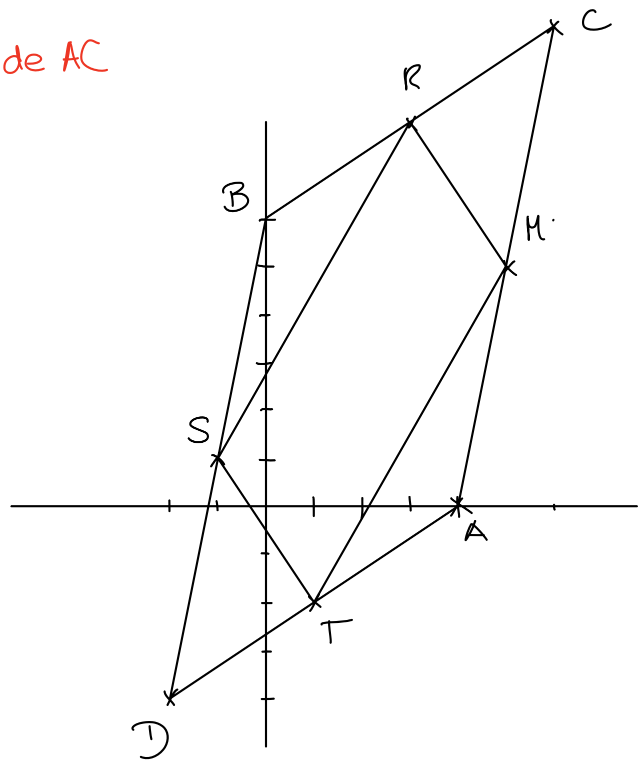
Ex 1.4.32

$$G(3;4) = \left(\frac{6-2+x}{3} ; \frac{-1+6+y}{3}\right) = \left(\frac{4+x}{3} ; \frac{5+y}{3}\right)$$

$$\Leftrightarrow \begin{cases} \frac{4+x}{3} = 3 & \mid \cdot 3 \\ \frac{5+y}{3} = 4 & \mid \cdot 3 \end{cases} \Leftrightarrow \begin{cases} 4+x = 9 \\ 5+y = 12 \end{cases} \Leftrightarrow \begin{cases} x = 5 \\ y = 7 \end{cases} \Rightarrow G(5;7)$$

Ex 1.4.33

⚠ donnée M milieu de AC



a) $M\left(\frac{4+6}{2} ; \frac{0+10}{2}\right) = M(5;5)$

$R\left(\frac{0+6}{2} ; \frac{6+10}{2}\right) = R(3;8)$

$S\left(\frac{0-2}{2} ; \frac{6-4}{2}\right) = S(-1;1)$

$T\left(\frac{4-2}{2} ; \frac{0-4}{2}\right) = T(1;-2)$

b) TMRS est un //-gramme

car $\vec{TM} = \begin{pmatrix} 5-1 \\ 5+2 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

et $\vec{SR} = \begin{pmatrix} 3+1 \\ 8-1 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ sont égaux.

c) Comme TMRS est un //-gramme les diagonales se coupent en leur milieu $\Rightarrow$ I est le milieu de TR (ou de MS)

$$\Rightarrow I\left(\frac{1+3}{2} ; \frac{-2+8}{2}\right) = I(2;3)$$