Rappel : $\boxed{(u^n)' = n\cdot u^{n-1}\cdot u'}$ dérivée interne

par exple : $\left[(3x^2+2x+1)^5\right]' = 5(3x^2+2x+1)^4\cdot(6x+2)$

On a
$$\boxed{\int u^n\cdot \underbrace{u'}_{\text{dérivée}}\,dx = \frac{1}{n+1}u^{n+1} + C \quad \text{avec } n \neq -1}$$

preuve : $\left(\frac{1}{n+1}u^{n+1}\right)' = \frac{1}{\cancel{n+1}}\cdot\cancel{(n+1)}u^{n+1-1}\cdot u' = u^n\cdot u'$

Rem ⚠ $\int u\cdot v\,dx \neq \int u\,dx\cdot\int v\,dx$ (primitive d'un produit pas dans ce cours)

Exemples

1) $\int (5x^2+2x+1)^3\cdot(10x+2)\,dx = \frac{1}{4}(5x^2+2x+1)^4 + C$

$u = 5x^2+2x+1$

$u' = 10x+2$

$$\int (5x^2+2x+1)^3\cdot(5x+1)dx = \frac{1}{2}\int (5x^2+2x+1)^3\underbrace{(5x+1)\cdot 2}_{u'}\,dx$$
$$= \frac{1}{2}\cdot\frac{1}{4}(5x^2+2x+1)^4 + C$$
$$= \frac{1}{8}(5x^2+2x+1)^4 + C$$

vérif : $\left(\frac{1}{8}(5x^2+2x+1)^4\right)' = \frac{1}{8}\cdot 4(5x^2+2x+1)^3(10x+2)$
$$= \frac{1}{2}(5x^2+2x+1)^3(10x+2)$$
$$= (5x^2+2x+1)^3(5x+1)$$

2) $\displaystyle\int \frac{1}{\sqrt{4x+1}}\,dx = \int (\underbrace{4x+1}_{})^{-\frac{1}{2}}\,dx = \frac{1}{4}\int (4x+1)^{-\frac{1}{2}}\cdot 4\,dx$

$u = 4x+1$

$u' = 4$

$$= \frac{1}{4}\cdot\frac{1}{\underbrace{-\frac{1}{2}+1}_{1/2}}(4x+1)^{\overbrace{-\frac{1}{2}+1}^{1/2}} + C$$

$$= \frac{1}{4}\cdot 2\,(4x+1)^{1/2} + C$$

$$= \frac{1}{2}\sqrt{4x+1} + C$$

3) $\displaystyle\int \frac{3x}{(x^2+2)^3}\,dx = \int 3x\cdot(\underbrace{x^2+2}_{u})^{-3}\,dx = \frac{3}{2}\int 2x\cdot(x^2+2)^{-3}\,dx$

$u = x^2+2$

$u' = 2x$

$$= \frac{3}{2}\cdot\frac{1}{\underbrace{-3+1}_{-2}}(x^2+2)^{\overbrace{-3+1}^{-2}} + C$$

$$= \frac{3}{2}\cdot\left(-\frac{1}{2}\right)(x^2+2)^{-2} + C$$

$$= -\frac{3}{4(x^2+2)^2} + C$$

4) $\int (x^2+x+1)\cdot(2x+1)\,dx = \frac{1}{2}(x^2+x+1)^2 + C$

$u = x^2+x+1$

$u' = 2x+1$

$\int (x^2+x+1)\,dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$