2.8.5 Déterminer, suivant les valeurs de l'entier naturel n, les asymptotes de la fonction f définie par $f(x) = \frac{x^n+3}{x^2-9}$. $= \frac{x^n+3}{x^2-9}$ $\quad n \in \mathbb{N} = \{0; 1; 2; \dots\}$

$ED(f) = \mathbb{R} - \{\pm 3\}$

$\underline{n=0}$ $\quad f(x) = \frac{4}{x^2-9}$

AV/trou : $\lim\limits_{x\to 3} f(x) = \text{"}\frac{4}{0}\text{"} = \infty \Rightarrow$ AV en $x=3$

$\lim\limits_{x\to -3} f(x) = \text{"}\frac{4}{0}\text{"} = \infty \Rightarrow$ AV en $x=-3$

AH/AO : $\lim\limits_{x\to\infty} \frac{4}{x^2} = \frac{4}{\infty} = 0 \Rightarrow$ AH en $y=0$

$\underline{n=1}$ $\quad f(x) = \frac{x+3}{x^2-9}$

AV/trou : $\lim\limits_{x\to 3} f(x) = \text{"}\frac{6}{0}\text{"} = \infty \Rightarrow$ AV en $x=3$

$\lim\limits_{x\to -3} f(x) = \text{"}\frac{0}{0}\text{"} = \lim\limits_{x\to -3} \frac{x+3}{(x+3)(x-3)} = \lim\limits_{x\to -3} \frac{1}{x-3} = \frac{1}{-3-3} = \frac{1}{-6} \Rightarrow$ "trou" $(-3; -\frac{1}{6})$

AH : $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{\cancel{x}}{x^{\cancel{2}}} = \lim\limits_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0 \Rightarrow$ AH en $y=0$

$\underline{n=2}$ $\quad f(x) = \frac{x^2+3}{x^2-9}$

AV : $\lim\limits_{x\to 3} f(x) = \text{"}\frac{12}{0}\text{"} = \infty \Rightarrow$ AV en $x=3$

$\lim\limits_{x\to -3} f(x) = \text{"}\frac{12}{0}\text{"} = \infty \Rightarrow$ AV en $x=-3$

AH : $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{\cancel{x^2}^{\,1}}{\cancel{x^2}_{\,1}} = 1 \Rightarrow$ AH en $y=1$

$\underline{n=3}$ $\quad f(x) = \frac{x^3+3}{x^2-9}$

AV : $x=3$ et $x=-3$

AO :

$$\begin{array}{r|l} x^3+3 & x^2-9 \\ \cline{2-2} -x^3+9x & x \\ \hline 9x+3 & \end{array}$$

$\Rightarrow y = x$ AO

$\underline{n>3}$ $\quad$ AV : $x=3$ et $x=-3$ et ni AH ni AO $\quad (\deg(N) > \deg(D)+1)$

$\underline{2.8.7}$ $\qquad f(x) = \dfrac{ax^2+bx}{x+c}$ $\qquad x=3 \quad y = x+2$ asymptotes

$c=-3$ car AV : $x=3$ $\Rightarrow$ 3 est une v.i.

$$\Rightarrow f(x) = \frac{ax^2+bx}{x-3}$$

$$m = 1 = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{ax^2+bx}{x^2-3x} = \lim_{x\to\infty} \frac{a\cancel{x^2}}{\cancel{x^2}} = a \Rightarrow f(x) = \frac{x^2+bx}{x-3}$$

$$h = 2 = \lim_{x\to\infty} (f(x) - x) = \lim_{x\to\infty} \frac{x^2+bx}{x-3} - x \overset{\text{"}\infty-\infty\text{"}}{=} \lim_{x\to\infty} \frac{x^2+bx-x(x-3)}{x-3}$$

$$= \lim_{x\to\infty} \frac{bx+3x}{x-3} = \lim_{x\to\infty} \frac{(b+3)x}{x-3} = \lim_{x\to\infty} \frac{(b+3)\cancel{x}}{\cancel{x}} = b+3 \Rightarrow b+3=2$$

$$b=-1$$

$$\Rightarrow f(x) = \frac{x^2-x}{x-3}$$