Problème 6 (14 points)

a) $(-1)^3 - 4 \cdot (-1) = (-1)^2 - 2 \cdot (-1) = -(-1) + 2 = 3 \quad \checkmark \quad \Rightarrow \quad A \in c_1 \cap c_2 \cap d$

$2^3 - 4 \cdot 2 = 2^2 - 2 \cdot 2 = -2 + 2 = 0 \quad \checkmark \quad \Rightarrow \quad B \in c_1 \cap c_2 \cap d$

b) $I_1 = \int_{-1}^{0} [-x + 2 - (x^3 - 4x)]\,\mathrm{d}x = \int_{-1}^{0} (-x^3 + 3x + 2)\,\mathrm{d}x = \left[-\frac{1}{4}x^4 + \frac{3}{2}x^2 + 2x\right]_{-1}^{0}$

$= 0 - \left(-\frac{1}{4} + \frac{3}{2} - 2\right) = \frac{3}{4}$

$I_2 = \int_{0}^{2} [-x + 2 - (x^2 - 2x)]\,\mathrm{d}x = \int_{0}^{2} (-x^2 + x + 2)\,\mathrm{d}x = \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right]_{0}^{2}$

$= -\frac{8}{3} + 2 + 4 - 0 = \frac{10}{3}$

$\Rightarrow \quad \text{aire de } D_1 = \frac{3}{4} + \frac{10}{3} = \boxed{\frac{49}{12}\ \mathrm{u}^2}$

c) $y = x^3 - 4x$ (fonction impaire)

$V = 2 \cdot \pi \int_{0}^{2} (x^3 - 4x)^2\,\mathrm{d}x = 2\pi \int_{0}^{2} (x^6 - 8x^4 + 16x^2)\,\mathrm{d}x = 2\pi \cdot \left[\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3\right]_{0}^{2}$

$= 2\pi \left(\frac{128}{7} - \frac{256}{5} + \frac{128}{3} - 0\right) = \boxed{\frac{2'048\pi}{105}\ \mathrm{u}^3}$

Corrigé 2013

Problème 1 (26 points)

a) $\ln(x+3)$ est défini si et seulement si $x+3>0$. Ainsi $\boxed{ED(f)=]-3;+\infty[.}$
f s'annule si $x=0$ ou si $\ln(x+3)=0 \Leftrightarrow x+3=1 \Leftrightarrow x=-2$.

b) $\boxed{ED(g)=\mathbb{R}.}$ $g(x)=\frac{3}{2}x\cdot(x+2)$. Ainsi, g s'annule en $x=0$ et en $x=-2$, comme f.
D'où $\boxed{I_1(-2;0)}$ et $\boxed{I_2(0;0).}$

c)
$$\begin{aligned}
F(x)' &= 2\left[(x^2-9)\ln(x+3)\right]' - [x^2]' + [6x]' \\
&= 2\left[2x\ln(x+3)+(x^2-9)\frac{1}{x+3}\right] - 2x+6 \\
&= 4x\ln(x+3)+2\frac{(x+3)(x-3)}{x+3} - 2x+6 \\
&= 4x\ln(x+3)+2(x-3)-2x+6 \\
&= 4x\ln(x+3) = f(x).
\end{aligned}$$

d)
$$\begin{aligned}
\int_{-2}^{0} g(x)-f(x)dx &= \left[\frac{1}{2}x^3+\frac{3}{2}x^2-2(x^2-9)\ln(x+3)+x^2-6x\right]_{-2}^{0} \\
&= 18\ln 3 - \left[\frac{1}{2}(-2)^3+\frac{3}{2}(-2)^2-2((-2)^2-9)\ln(-2+3)+(-2)^2-6(-2)\right] \\
&= 18\ln 3-[-4+6-0+4+12] = 18\ln 3-18 = \boxed{18(\ln 3-1)} \quad (\simeq 1.77502).
\end{aligned}$$

e)
$$\begin{aligned}
\pi\int_{-2}^{0} g^2(x)dx &= \pi\int_{-2}^{0}\left[\frac{3}{2}x^2+3x\right]^2 dx \\
&= \pi\int_{-2}^{0}\left[\frac{9}{4}x^4+9x^3+9x^2\right]dx \\
&= \pi\left[\frac{9}{20}x^5+\frac{9}{4}x^4+3x^3\right]_{-2}^{0} \\
&= \pi\left[0-\frac{9}{4\cdot 5}(-2)^5-\frac{9}{4}(-2)^4-3(-2)^3\right] \\
&= \pi\left[\frac{72}{5}-36+24\right] = \boxed{\frac{12}{5}\pi}
\end{aligned}$$

Corrigé 2014

Problème 2 corrigé (14 points)

a)

$$\text{Aire} = 2\int_0^3 (-x^2+15)\,dx = 2\left[-\frac{1}{3}x^3+15x\right]_0^3 = 2\left[-\frac{1}{3}3^3+15\cdot 3\right] = 2(-9+45) = 2\cdot 36 = \boxed{72\,u^2}$$

Variante :

$$\text{Aire} = \int_{-3}^3 (-x^2+15)\,dx = \left[-\frac{1}{3}x^3+15x\right]_{-3}^3 = \left[-\frac{1}{3}3^3+15\cdot 3\right] - \left[-\frac{1}{3}(-3)^3+15\cdot(-3)\right] =$$

$$(-9+45)-(9-45) = 36-(-36) = 36+36 = \boxed{72\,u^2}$$

b)

$$\text{Volume} = 2\pi\int_0^3 (-x^2+a)^2 dx = 2\pi\int_0^3 (x^4-2ax^2+a^2)\,dx = 2\pi\left[\frac{1}{5}x^5-2a\frac{1}{3}x^3+a^2x\right]_0^3 =$$

$$2\pi\left[\frac{1}{5}3^5-2a\frac{1}{3}3^3+a^2 3\right] = 2\pi\left[\frac{243}{5}-18a+3a^2\right] = 2\pi\left[\frac{243}{5}-\frac{90a}{5}+\frac{15a^2}{5}\right] =$$

$$\frac{2\pi}{5}(15a^2-90a+243) = \frac{6\pi}{5}(5a^2-30a+81)$$

Variante :

$$\text{Volume} = \pi\int_{-3}^3 (-x^2+a)^2 dx = \pi\int_{-3}^3 (x^4-2ax^2+a^2)\,dx = \pi\left[\frac{1}{5}x^5-2a\frac{1}{3}x^3+a^2x\right]_{-3}^3 =$$

$$\pi\cdot\left\{\left[\frac{1}{5}3^5-2a\frac{1}{3}3^3+a^2 3\right]-\left[\frac{1}{5}(-3)^5-2a\frac{1}{3}(-3)^3+a^2(-3)\right]\right\} =$$

$$\pi\cdot\left\{\left[\frac{243}{5}-18a+3a^2\right]-\left[-\frac{243}{5}+18a-3a^2\right]\right\} = \pi\left[\frac{243}{5}-\frac{90a}{5}+\frac{15a^2}{5}+\frac{243}{5}-\frac{90a}{5}+\frac{15a^2}{5}\right] =$$

$$\frac{\pi}{5}(30a^2-180a+486) = \frac{6\pi}{5}(5a^2-30a+81)$$

Donc $\dfrac{6\pi}{5}(5a^2-30a+81) = \dfrac{14'736\,\pi}{5} \Rightarrow 5a^2-30a+81 = 2456 \Rightarrow 5a^2-30a-2375 = 0$

$\Rightarrow a^2-6a-475 = 0 \Rightarrow (a-25)(a+19) = 0 \Rightarrow a = 25$ et $a = -19 \overset{a>9}{\Rightarrow} \boxed{a = 25}$

Problème 2 (21 points)

a) $\dfrac{2}{2x-3} = 2x-2 \quad \Rightarrow \quad 2=(2x-3)(2x-2) \quad \Rightarrow \quad 2 = 4x^2-10x+6$

$\Rightarrow \quad 4x^2-10x+4=0 \quad \Rightarrow 2x^2-5x+2=0 \quad \Rightarrow \quad (2x-1)(x-2)=0$

$\Rightarrow \quad (2;2)$ et $\left(\frac{1}{2};-1\right) \leftarrow$ point à éliminer $\Rightarrow$ $\boxed{A(2;2)}$

b) aire du triangle sous la droite d (coupe l'axe Ox en $x=1$) : $\dfrac{1\cdot 2}{2} = 1$ u^2

ou alors : $\displaystyle\int_1^2 (2x-2)\,dx = x^2-2x\Big|_1^2 = 4-4-(1-2) = 1$ u^2

aire sous la courbe c : $\displaystyle\int_2^4 \frac{2}{2x-3}\,dx = \ln(|2x-3|)\Big|_2^4 = \ln(5) - \underbrace{\ln(1)}_{0} = \ln(5)$ u^2

$\Rightarrow \quad \boxed{A = \ln(5)+1\ u^2}$

c) volume du cône engendré par la rotation de d : $\dfrac{\pi\cdot 2^2\cdot 1}{3} = \dfrac{4\pi}{3}$ u^3

ou alors : $\displaystyle\pi\int_1^2 (2x-2)^2\,dx = \pi\int_1^2 (4x^2-8x+4)\,dx = \pi\left[\frac{4}{3}x^3-4x^2+4x\right]_1^2$

$\displaystyle = \pi\left[\frac{32}{3}-16+8-\left(\frac{4}{3}-4+4\right)\right] = \pi\left[\frac{28}{3}-8\right] = \frac{4\pi}{3}$ u^3

ou encore : $\displaystyle\pi\int_1^2 (2x-2)^2\,dx = \frac{\pi}{2}\int_1^2 2(2x-2)^2\,dx = \frac{\pi}{2}\left[\frac{1}{3}(2x-2)^3\right]_1^2$

$\displaystyle = \frac{\pi}{2}\left[\frac{8}{3}-0\right] = \frac{4\pi}{3}$ u^3

volume engendré par la rotation de c : $\displaystyle\pi\int_2^4 \frac{4}{(2x-3)^2}\,dx = 2\pi\int_2^4 2(2x-3)^{-2}\,dx$

$\displaystyle = 2\pi\left[\frac{1}{(-1)}(2x-3)^{-1}\right]_2^4 = 2\pi\left[\frac{-1}{2x-3}\right]_2^4 = 2\pi\underbrace{\left[-\frac{1}{5}-(-1)\right]}_{\frac{4}{5}} = \frac{8\pi}{5}$ u^3

$\Rightarrow \quad V = \dfrac{4\pi}{3}+\dfrac{8\pi}{5} = \boxed{\dfrac{44\pi}{15}\ \text{u}^3}$

Problème 3 (16 points)

a) $\displaystyle\int_0^6 -x^2+7x\,\mathrm{d}x = -\frac{1}{3}x^3+\frac{7}{2}x^2\Big|_0^6 = [-\frac{1}{3}\cdot 6^3+\frac{7}{2}\cdot 6^2]-[0] = -\frac{216}{3}+7\cdot 18 = -72+126 = 54\,\mathrm{u}^2.$

$\displaystyle\int_2^6 \sqrt{x-2}\,\mathrm{d}x = \int_2^6 (x-2)^{\frac{1}{2}}\,\mathrm{d}x = \frac{1}{\frac{1}{2}+1}(x-2)^{\frac{1}{2}+1}\Big|_2^6 = \frac{2}{3}\cdot(x-2)^{\frac{3}{2}}\Big|_2^6 = \frac{2}{3}\cdot\sqrt{(x-2)^3}\Big|_2^6 =$

$\displaystyle\frac{2}{3}\cdot\sqrt{(6-2)^3}-\frac{2}{3}\cdot\sqrt{(2-2)^3} = \frac{2}{3}\cdot\sqrt{4^3} = \frac{2}{3}\cdot 8 = \frac{16}{3}\,\mathrm{u}^2 = 5{,}\overline{3}\,\mathrm{u}^2.$

Aire du domaine D : $\displaystyle 54-\frac{16}{3} = \frac{162}{3}-\frac{16}{3} = \boxed{\frac{146}{3}\,\mathrm{u}^2} = \boxed{48{,}\overline{6}\,\mathrm{u}^2}.$

b) $\displaystyle\pi\int_0^6 (-x^2+7x)^2\,\mathrm{d}x = \pi\int_0^6 x^4-14x^3+49x^2\,\mathrm{d}x = \pi[\frac{1}{5}x^5-14\cdot\frac{1}{4}x^4+49\cdot\frac{1}{3}x^3]\Big|_0^6 =$

$\displaystyle\pi\{[\frac{1}{5}\cdot 6^5-\frac{7}{2}\cdot 6^4+\frac{49}{3}\cdot 6^3]-[0]\} = \pi[\frac{7'776}{5}-4'536+3'528] = \pi\frac{7'776-22'680+17'640}{5} =$

$\displaystyle\pi\frac{2'736}{5} = \frac{2'736}{5}\pi\,\mathrm{u}^3 = 547{,}2\,\pi\,\mathrm{u}^3.$

$\displaystyle\pi\int_2^6 (\sqrt{x-2})^2\,\mathrm{d}x = \pi\int_2^6 x-2\,\mathrm{d}x = \pi[\frac{1}{2}x^2-2x]\Big|_2^6 = \pi[(\frac{1}{2}\cdot 6^2-12)-(\frac{1}{2}\cdot 2^2-4)] =$

$\pi[6+2] = 8\pi\,\mathrm{u}^3.$

Volume du solide de révolution : $\displaystyle\frac{2'736}{5}\pi-\frac{40}{5}\pi = \boxed{\frac{2'696}{5}\pi\,\mathrm{u}^3} = \boxed{539{,}2\,\pi\,\mathrm{u}^3}.$