

Ex 1.3.3

u

$$a) \int 3 dx = 3x + C$$

$$b) \int 5x dx = \frac{5}{2}x^2 + C$$

$$c) \int (2x+1) dx = x^2 + x + C$$

$$d) \int (5x-4) dx = \frac{5}{2}x^2 - 4x + C$$

$$e) \int (2x^2-3x+2) dx = \frac{2}{3}x^3 - \frac{3}{2}x^2 + 2x + C$$

$$f) \int 5x^3 dx = \frac{5}{4}x^4 + C$$

$$g) \int (-3x^4) dx = -\frac{3}{5}x^5 + C$$

$$h) \int (3x^5+2x^4-1) dx = \frac{3}{6}x^6 + \frac{2}{5}x^5 - x + C = \frac{1}{2}x^6 + \frac{2}{5}x^5 - x + C$$

Ex 1.3.4

$$a) \int \frac{dx}{x^2} = \int x^{-2} dx = \frac{1}{-1} x^{-1} + C = -\frac{1}{x} + C$$

$$b) \int \frac{2dx}{x^3} = \int 2x^{-3} dx = 2 \cdot \frac{1}{-2} x^{-2} + C = -\frac{1}{x^2} + C$$

$$c) \int \frac{-7dx}{x^5} = \int -7x^{-5} dx = -7 \cdot \frac{1}{-4} x^{-4} + C = \frac{7}{4x^4}$$

$$d) \int \sqrt{x} dx = \int x^{1/2} dx = \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + C = \frac{2}{3} \sqrt{x^3} + C$$

$$e) \int \sqrt[3]{x} dx = \int x^{1/3} dx = \frac{1}{\frac{4}{3}} x^{\frac{4}{3}} + C = \frac{3}{4} \sqrt[3]{x^4} + C$$

$$f) \int \frac{dx}{\sqrt{x}} = \int x^{-1/2} dx = \frac{1}{\frac{1}{2}} x^{1/2} + C = 2\sqrt{x} + C$$

$$g) \int \frac{dx}{\sqrt[3]{x^2}} = \int x^{-2/3} dx = \frac{1}{\frac{1}{3}} x^{1/3} + C = 3\sqrt[3]{x} + C$$

Ex 1.3.5

$$c) \int (x+3)^3 dx = \frac{1}{4} (x+3)^4 + C$$

$$u = x+3$$

$$u' = 1$$

$$d) \frac{1}{2} \int 2(2x-1)^2 dx = \frac{1}{2} \cdot \frac{1}{3} (2x-1)^3 + C = \frac{1}{6} (2x-1)^3 + C$$

$$u = 2x-1$$

$$u' = 2$$

$$e) \frac{1}{7} \int 7(7x-2)^5 dx = \frac{1}{7} \cdot \frac{1}{6} (7x-2)^6 + C = \frac{1}{42} (7x-2)^6 + C$$

$$u = 7x-2$$

$$u' = 7$$

$$f) \int (3x^2+x)^3 (6x+1) dx = \frac{1}{4} (3x^2+x)^4 + C$$

$$u = 3x^2+x$$

$$u' = 6x+1$$

$$g) \frac{1}{8} \int (4x^2+3)^4 \cdot 8 dx = \frac{1}{8} \cdot \frac{1}{5} (4x^2+3)^5 + C = \frac{1}{40} (4x^2+3)^5 + C$$

$$u = 4x^2+3$$

$$u' = 8x$$

$$j) \int \sqrt{x+3} dx = \int (x+3)^{1/2} dx = \frac{1}{3/2} (x+3)^{3/2} + C = \frac{2}{3} \sqrt{(x+3)^3} + C$$

$$u = x+3$$

$$u' = 1$$

$$k) \frac{1}{3} \int \frac{3 \cdot 1}{\sqrt{3x+1}} dx = \frac{1}{3} \int 3(3x+1)^{-1/2} dx = \frac{1}{3} \cdot \frac{1}{1/2} (3x+1)^{1/2} + C = \frac{2}{3} \sqrt{3x+1} + C$$

$$u = 3x+1$$

$$u' = 3$$

$$l) \frac{1}{2} \int \frac{(x+1) \cdot 2}{\sqrt{x^2+2x}} dx = \frac{1}{2} \int (x^2+2x)^{-1/2} \cdot 2(x+1) dx = \frac{1}{2} \cdot \frac{1}{1/2} (x^2+2x)^{1/2} + C = \sqrt{x^2+2x} + C$$

$$u = x^2+2x$$

$$u' = 2x+2 = 2(x+1)$$

Ex 1.3.6

$$a) \int (3x^2 - 2x + 3) dx = \underline{x^3 - x^2 + 3x + C}$$

$$b) \int \frac{3x^4 - 3x^2 - 7}{4x^2} dx \quad \deg(N) \geq \deg(D) \Rightarrow \text{division euclidienne}$$

ici on n'est pas obligé de la poser car le dénominateur est un monôme, on peut diviser directement.

$$= \int \left(\frac{3x^4}{4x^2} - \frac{3x^2}{4x^2} - \frac{7}{4x^2} \right) dx$$

$\downarrow \Delta$ $\frac{7}{4} \cdot \frac{1}{x^2} = \frac{7}{4} \cdot x^{-2} (\neq 7 \cdot 4x^{-2})$

$$= \int \left(\frac{3x^2}{4} - \frac{3}{4} - \frac{7}{4} \cdot x^{-2} \right) dx$$
$$= \frac{3}{4} \cdot \frac{1}{3} x^3 - \frac{3}{4} x - \frac{7}{4} \cdot \frac{1}{-1} x^{-1} + C = \underline{\underline{\frac{1}{4}x^3 - \frac{3}{4}x + \frac{7}{4x} + C}}$$

$$c) \int 7\sqrt[4]{x^3} dx = \int 7x^{3/4} dx = 7 \cdot \frac{1}{\frac{7}{4}} x^{7/4} + C = \underline{\underline{4\sqrt[4]{x^7} + C}}$$

$$d) \int (\sqrt{x} - \sqrt[3]{x}) dx = \int (x^{1/2} - x^{1/3}) dx = \frac{1}{\frac{3}{2}} x^{3/2} - \frac{1}{\frac{4}{3}} x^{4/3} + C$$
$$= \underline{\underline{\frac{2}{3}\sqrt{x^3} - \frac{3}{4}\sqrt[3]{x^4} + C}}$$

$$i) \int (3x^2 - 7)^2 dx = \int (9x^4 - 42x^2 + 49) dx$$
$$= \frac{9}{5}x^5 - \frac{42}{3}x^3 + 49x + C = \underline{\underline{\frac{9}{5}x^5 - 14x^3 + 49x + C}}$$

Δ ne pas confondre avec $\int \underbrace{(3x^2 - 7)^2}_{u^2} \cdot \underbrace{6x}_{u'} dx = \frac{1}{3}(3x^2 - 7)^2 + C$

Un des facteurs n'est pas la dérivée interne de l'autre $\Rightarrow$ on multiplie après transformation.

$$j) \int \sqrt{x} \cdot (x^2 - 5) dx = \int x^{\frac{1}{2}} (x^2 - 5) dx = \int (x^{\frac{1}{2}+2} - 5x^{\frac{1}{2}}) dx = \int (x^{\frac{5}{2}} - 5x^{\frac{1}{2}}) dx$$
$$= \frac{1}{\frac{5}{2}+1} x^{\frac{5}{2}+1} - 5 \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + C$$
$$= \frac{2}{7} x^{\frac{7}{2}} - \frac{10}{3} x^{\frac{3}{2}} + C$$
$$= \underline{\underline{\frac{2}{7}\sqrt{x^7} - \frac{10}{3}\sqrt{x^3} + C}}$$

$$k) \quad \frac{1}{3} \int 3(3x-5)^6 dx = \frac{1}{3} \cdot \frac{1}{7} (3x-5)^7 + C = \underline{\underline{\frac{1}{21} (3x-5)^7 + C}}$$

$$l) \quad \int \frac{\overbrace{12}^{u'} \underbrace{(-3)(-4)}}{(4-3x)^4} dx = -4 \int (-3) \cdot (4-3x)^{-4} dx = -4 \cdot \frac{1}{-3} (4-3x)^{-3} + C = \underline{\underline{\frac{4}{3(4-3x)^3} + C}}$$

$$u = 4-3x$$

$$u' = -3$$

$$m) \quad \int \sqrt[3]{(3x-8)^2} dx = \frac{1}{3} \int 3(3x-8)^{2/3} dx = \frac{1}{3} \cdot \frac{1}{\frac{5}{3}} (3x-8)^{\frac{5}{3}} + C$$

$$u = 3x-8$$

$$u' = 3$$

$$= \underline{\underline{\frac{1}{5} \sqrt[3]{(3x-8)^5} + C}}$$

$$o) \quad \int x \sqrt{x^2+1} dx = \frac{1}{2} \int 2x (x^2+1)^{1/2} dx = \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (x^2+1)^{\frac{3}{2}} + C$$

$$u = x^2+1$$

$$u' = 2x$$

$$= \underline{\underline{\frac{1}{3} \sqrt{(x^2+1)^3} + C}}$$

$$p) \quad \int \frac{2x-1}{\sqrt{x^2-x-1}} dx = \int (x^2-x-1)^{-\frac{1}{2}} (2x-1) dx = \frac{1}{\frac{1}{2}} (x^2-x-1)^{\frac{1}{2}} + C$$

$$u = x^2-x-1$$

$$u' = 2x-1$$

$$= \underline{\underline{2\sqrt{x^2-x-1} + C}}$$

Ex 1.3.10

$$\begin{aligned} \text{a) } \int_1^4 (x^2 - 2x + 3) dx &= \left. \frac{1}{3}x^3 - x^2 + 3x \right|_1^4 \\ &= \left(\frac{1}{3} \cdot 64 - 16 + 12 \right) - \left(\frac{1}{3} - 1 + 3 \right) \\ &= \frac{63}{3} - 6 = 21 - 6 = \underline{15} \end{aligned}$$

$$\begin{aligned} \text{b) } \int_{-1}^1 (2x^3 + 3x^2 + 2x - 1) dx &= \left. \frac{1}{2}x^4 + x^3 + x^2 - x \right|_{-1}^1 \\ &= \left(\frac{1}{2} + 1 + 1 - 1 \right) - \left(\frac{1}{2} - 1 + 1 + 1 \right) = \underline{0} \end{aligned}$$

$$\begin{aligned} \text{c) } \int_{-1}^1 \sqrt[3]{x+1} dx &= \int_{-1}^1 (x+1)^{1/3} dx = \left. \frac{1}{\frac{4}{3}} (x+1)^{4/3} \right|_{-1}^1 \\ &= \frac{3}{4} \left(2^{4/3} - 0^{4/3} \right) = \frac{3}{4} \sqrt[3]{16} = \frac{3}{4} \cdot 2 \sqrt[3]{2} \\ &= \underline{\frac{3}{2} \sqrt[3]{2}} \end{aligned}$$

$$\begin{aligned} \text{f) } \int_0^2 (1+2x)^3 dx &= \frac{1}{2} \int_0^2 (1+2x)^3 \cdot 2 dx = \frac{1}{2} \cdot \frac{1}{4} (1+2x)^4 \Big|_0^2 \\ &\quad \begin{matrix} u=1+2x \\ u'=2 \end{matrix} \\ &= \frac{1}{8} 5^4 - \frac{1}{8} \cdot 1^4 = \frac{1}{8} (625 - 1) = \frac{624}{8} = \underline{78} \end{aligned}$$

$$\begin{aligned} \text{g) } \int_1^4 \frac{dx}{\sqrt{x}} &= \int_1^4 x^{-1/2} dx = \left. \frac{1}{\frac{1}{2}} x^{1/2} \right|_1^4 = 2\sqrt{x} \Big|_1^4 = 2 \cdot 2 - 2 \cdot 1 = \underline{2} \end{aligned}$$

Ex 1.3.16

a) $f(x) = 9 - x^2$ (parabole $\cap$)

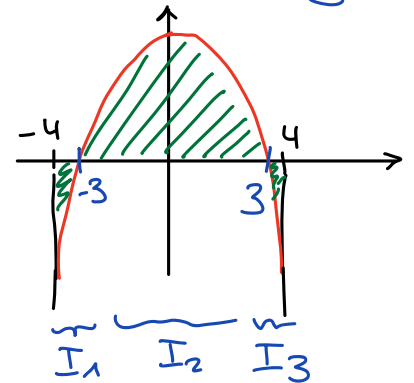
$a = -4$ et $b = 4$

$ED(f) = \mathbb{R}$

zéros : $9 - x^2 = 0 \Leftrightarrow (3+x)(3-x) = 0 \Leftrightarrow x = \begin{matrix} -3 \\ 3 \end{matrix}$

signe :

x	-4	-3	3	4	
$\text{sgn}(f)$	-	0	+	0	-
	I_1		I_2	I_3	



$$I_1 = \int_{-4}^{-3} (9 - x^2) dx = \left[9x - \frac{1}{3}x^3 \right]_{-4}^{-3} \\ = (-27 + 9) - \left(-36 + \frac{64}{3} \right) = -\frac{10}{3}$$

$$I_2 = \int_{-3}^3 (9 - x^2) dx = \left[9x - \frac{1}{3}x^3 \right]_{-3}^3 \\ = (27 - 9) - (-27 + 9) = 36$$

$$I_3 = \int_3^4 (9 - x^2) dx = \left[9x - \frac{1}{3}x^3 \right]_3^4 \\ = \left(36 - \frac{64}{3} \right) - (27 - 9) = -\frac{10}{3}$$

$$\Rightarrow A = \left| -\frac{10}{3} \right| + 36 + \left| -\frac{10}{3} \right| = 36 + 2 \cdot \frac{10}{3} = \underline{\underline{\frac{128}{3} \text{ u}^2}}$$

$$b) \quad f(x) = \frac{4}{x^2} - 1 \quad a=1, b=4$$

$$f(x) = \frac{4-x^2}{x^2} = \frac{(2-x)(2+x)}{x^2} \quad \leftarrow \text{zéros: } \pm 2$$

$$\leftarrow \text{v.i.: } 0 \quad (2)$$

$$ED(f) = \mathbb{R}^*$$

$$\text{signe: } \begin{array}{c|cccccc} x & -2 & 0 & 1 & 2 & 4 \\ \hline \text{sgn}(f) & - & 0 & + & + & 0 & - \\ & & & (2) & \underbrace{+} & \underbrace{0} & \\ & & & & I_1 & I_2 & \end{array} \quad \nwarrow f(1000): \frac{-}{+}$$

$$\begin{aligned} I_1 &= \int_1^2 \left(\frac{4}{x^2} - 1 \right) dx = \int_1^2 (4x^{-2} - 1) dx = 4 \cdot \frac{1}{-1} x^{-1} - x \Big|_1^2 \\ &= -\frac{4}{x} - x \Big|_1^2 = (-2-2) - (-4-1) = 1 \end{aligned}$$

$$I_2 = \int_2^4 \left(\frac{4}{x^2} - 1 \right) dx = -\frac{4}{x} - x \Big|_2^4 = (-1-4) - (-2-2) = -1$$

$$\Rightarrow \mathcal{A} = 1 + |-1| = 1 + 1 = \underline{2u^2}$$

$$d) f(x) = \sqrt{2x-4} \quad a=2, b=10$$

$$ED(f) = [2; +\infty[$$

$$\text{car } 2x-4 \geq 0$$

$$\begin{array}{c|cc} x & 2 & 10 \\ \hline & - & 0 & + \end{array}$$

$$A = \int_2^{10} \sqrt{2x-4} dx = \int_2^{10} (2x-4)^{1/2} dx = \frac{1}{2} \int_2^{10} (2x-4)^{1/2} \cdot 2 dx$$

$\begin{matrix} u=2x-4 \\ u'=2 \end{matrix}$

$$= \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (2x-4)^{3/2} \Big|_2^{10} = \frac{1}{3} \sqrt{(2x-4)^3} \Big|_2^{10}$$

$$= \frac{1}{3} 4^3 - \frac{1}{3} \cdot 0 = \underline{\underline{\frac{64}{3}}}$$

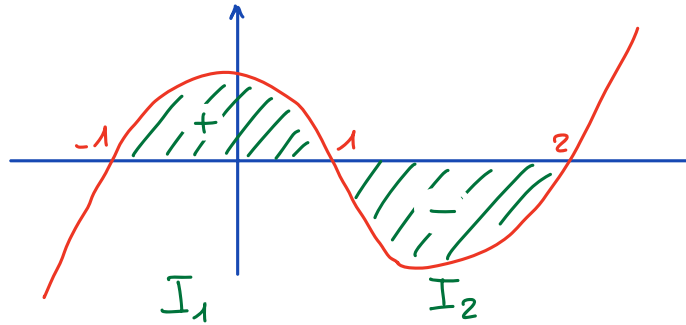
Ex 1.3.17

$$a) f(x) = x^3 - 2x^2 - x + 2 = x^2(x-2) - 1(x-2) = (x-2)(x^2-1) = (x-2)(x+1)(x-1)$$

$$ED(f) = \mathbb{R} \quad \text{zéros: } -1, 1 \text{ et } 2$$

$$\text{signe: } \begin{array}{c|cccc} x & -1 & 1 & 2 & \\ \hline \text{sgn}(f) & - & 0 & + & 0 & - & 0 & + \end{array} \quad f(+\infty): +$$

croquis:



$$\begin{aligned} I_1 &= \int_{-1}^1 (x^3 - 2x^2 - x + 2) dx = \left. \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x \right|_{-1}^1 \\ &= \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2 \right) - \left(\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2 \right) = \frac{8}{3} \end{aligned}$$

$$\begin{aligned} I_2 &= \int_1^2 f(x) dx = \left. \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x \right|_1^2 \\ &= \left(4 - \frac{16}{3} - 2 + 4 \right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2 \right) = -\frac{5}{12} \end{aligned}$$

$$\Rightarrow A = \frac{8}{3} + \frac{5}{12} = \underline{\underline{\frac{37}{12} u^2}}$$

b) $f(x) = x\sqrt{4-x^2}$ cond: $4-x^2 \geq 0 \Leftrightarrow (2+x)(2-x) \geq 0$ $\frac{x}{4-x^2} \begin{array}{c|cc} -2 & 2 \\ \hline - & 0 & + & 0 & - \end{array}$

ED(f) = $[-2; 2]$

zéros: $-2; 0; 2$

signe: $\frac{x}{\text{sgn}(f)} \begin{array}{c|ccc} -2 & 0 & 2 \\ \hline // & - & 0 & + & 0 & // \end{array}$
 $\underbrace{\quad}_{I_1} \quad \underbrace{\quad}_{I_2}$

$I_1 = \int_{-2}^0 x\sqrt{4-x^2} dx = \int_{-2}^0 x(4-x^2)^{1/2} dx = -\frac{1}{2} \int_{-2}^0 -2x(4-x^2)^{1/2} dx$
 $u = 4-x^2$
 $u' = -2x$

$= -\frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (4-x^2)^{3/2} \Big|_{-2}^0 = -\frac{1}{3} \sqrt{(4-x^2)^3} \Big|_{-2}^0$

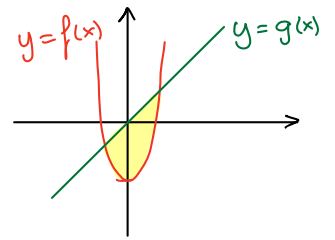
$= -\frac{1}{3} 8 - \left(-\frac{1}{3} \cdot 0\right) = -\frac{8}{3}$

$I_2 = \int_0^2 x\sqrt{4-x^2} dx = -\frac{1}{3} \sqrt{(4-x^2)^3} \Big|_0^2 = 0 - \left(-\frac{8}{3}\right) = \frac{8}{3}$

$\Rightarrow \mathcal{A} = \frac{8}{3} + \frac{8}{3} = \underline{\underline{\frac{16}{3}}}$ u^2

Ex 1.3.13

a) $f(x) = x^2 - 3$ et $g(x) = 2x$



pts $d'N$: $x^2 - 3 = 2x \Leftrightarrow x^2 - 2x - 3 = 0 \Leftrightarrow (x-3)(x+1) = 0$

$\downarrow$ $\downarrow$
 3 -1

$$\int_{-1}^3 (x^2 - 3 - 2x) dx = \int_{-1}^3 (x^2 - 2x - 3) dx = \left[\frac{1}{3}x^3 - x^2 - 3x \right]_{-1}^3$$

$$= (9 - 9 - 9) - \left(-\frac{1}{3} - 1 + 3\right) = -9 - \frac{5}{3} = -\frac{32}{3}$$

$\Rightarrow A = \left| -\frac{32}{3} \right| = \underline{\underline{\frac{32}{3} u^2}}$

b) $f(x) = x^2$ et $g(x) = 8 - x^2$

pts $d'N$: $x^2 = 8 - x^2 \Leftrightarrow 2x^2 = 8 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$

$$\int_{-2}^2 (x^2 - (8 - x^2)) dx = \int_{-2}^2 (2x^2 - 8) dx = \left[\frac{2}{3}x^3 - 8x \right]_{-2}^2$$

$$= \left(\frac{16}{3} - 16\right) - \left(-\frac{16}{3} + 16\right) = \frac{32}{3} - 32 = -\frac{64}{3}$$

$\Rightarrow A = \left| -\frac{64}{3} \right| = \underline{\underline{\frac{64}{3} u^2}}$

c) $f(x) = x^3 - 5x^2 + 6x$ et $g(x) = x^3 - 7x^2 + 12x$

pts $d'N$: $x^3 - 5x^2 + 6x = x^3 - 7x^2 + 12x \Leftrightarrow 2x^2 - 6x = 0 \Leftrightarrow 2x(x-3) = 0$

$\downarrow$ $\downarrow$
 0 3

$$\Rightarrow \int_0^3 (x^3 - 5x^2 + 6x - (x^3 - 7x^2 + 12x)) dx = \int_0^3 (2x^2 - 6x) dx$$

$$= \left[\frac{2}{3}x^3 - 3x^2 \right]_0^3 = \left(\frac{2}{3} \cdot 27 - 3 \cdot 9\right) - 0 = 18 - 27 = -9$$

$\Rightarrow A = |-9| = \underline{\underline{9 u^2}}$

$$d) \quad f(x) = x(6-2x^2) = -2x^3 + 6x \quad g(x) = x(2-x^2) = -x^3 + 2x$$

$$\begin{aligned} \text{pts d'I} : \quad -2x^3 + 6x &= -x^3 + 2x \Leftrightarrow -x^3 + 4x = 0 \Leftrightarrow -x(x^2 - 4) = 0 \\ &\Leftrightarrow -x(x+2)(x-2) = 0 \\ &\quad \downarrow \quad \downarrow \quad \downarrow \\ &\quad 0 \quad -2 \quad 2 \end{aligned}$$

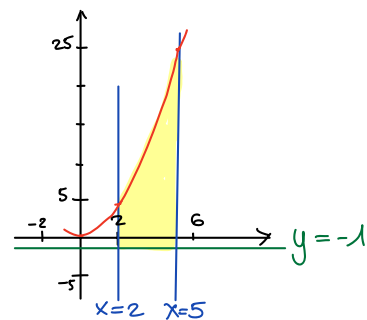
$$\begin{aligned} I_1 &= \int_{-2}^0 (f(x) - g(x)) dx = \int_{-2}^0 (-x^3 + 4x) dx = \left. -\frac{1}{4}x^4 + 2x^2 \right|_{-2}^0 \\ &= 0 - \left(-\frac{1}{4} \cdot 16 + 2 \cdot 4 \right) = +4 - 8 = -4 \end{aligned}$$

$$I_2 = \left. -\frac{1}{4}x^4 + 2x^2 \right|_0^2 = -\frac{1}{4} \cdot 16 + 2 \cdot 4 = -4 + 8 = 4$$

$$\Rightarrow \mathcal{A} = |-4| + 4 = \underline{8 \text{ u}^2}$$

Ex 1.3.21

$$y = x^2, \quad y = -1, \quad \underbrace{x=2 \text{ et } x=5}_{\text{bornes}}$$



$$\begin{aligned} \mathcal{A} &= \int_2^5 (x^2 - (-1)) dx = \int_2^5 (x^2 + 1) dx = \left. \frac{1}{3}x^3 + x \right|_2^5 \\ &= \left(\frac{1}{3} \cdot 125 + 5 \right) - \left(\frac{1}{3} \cdot 8 + 2 \right) = \frac{140}{3} - \frac{14}{3} = \underline{42 \text{ u}^2} \end{aligned}$$

Variante : Aire sous la courbe $y = x^2$ + Aire "sous" la courbe $y = -1$

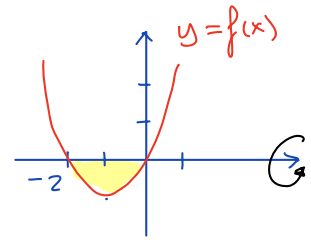
$$A_1 = \int_2^5 x^2 dx = \left. \frac{1}{3}x^3 \right|_2^5 = \frac{125}{3} - \frac{8}{3} = 39$$

$$A_2 = 1 \cdot 3 = 3 \quad (\text{aire d'un rectangle})$$

$$\Rightarrow \mathcal{A} = 39 + 3 = 42 \text{ u}^2$$

Ex 1.3.24

a) $f(x) = x^2 + 2x = x(x+2) \Rightarrow$ zéros: -2 et 0



$$V = \pi \int_{-2}^0 (x^2 + 2x)^2 dx = \pi \int_{-2}^0 (x^4 + 4x^3 + 4x^2) dx$$

$$= \pi \left(\frac{1}{5} x^5 + x^4 + \frac{4}{3} x^3 \right) \Big|_{-2}^0 = \pi \left(0 - \left(-\frac{32}{5} + 16 - \frac{32}{3} \right) \right) = \underline{\underline{\frac{16\pi}{15} u^3}}$$

b) $f(x) = \sqrt{1-x^2}$ zéros: $1-x^2 = 0 \Leftrightarrow (1+x)(1-x) = 0 \Rightarrow -1$ et 1

$$V = \pi \int_{-1}^1 (\sqrt{1-x^2})^2 dx = \pi \int_{-1}^1 (1-x^2) dx = \pi \left(x - \frac{1}{3} x^3 \right) \Big|_{-1}^1$$

$$= \pi \left(1 - \frac{1}{3} - \left(-1 + \frac{1}{3} \right) \right) = \pi \left(\frac{2}{3} - \left(-\frac{2}{3} \right) \right) = \underline{\underline{\frac{4\pi}{3} u^3}}$$

↑ Volume d'une sphère de rayon = 1 ($V = \frac{4}{3}\pi r^3$)

$$y = f(x) \Leftrightarrow y = \sqrt{1-x^2}$$

$$\Leftrightarrow y^2 = 1-x^2$$

$$\Leftrightarrow x^2 + y^2 = 1$$

équation d'un cercle de rayon = 1

Ex 1.3.26

a) $f(x) = x+1$ $a=1$ $b=3$

$$V = \pi \int_1^3 (x+1)^2 dx = \pi \int_1^3 (x^2 + 2x + 1) dx = \pi \left(\frac{1}{3}x^3 + x^2 + x \right) \Big|_1^3$$

$$= \pi \left(9 + 9 + 3 - \left(\frac{1}{3} + 1 + 1 \right) \right) = \pi \left(21 - \frac{7}{3} \right) = \underline{\underline{\frac{56\pi}{3} u^3}}$$

Variante: $\pi \int_1^3 (x+1)^2 dx = \pi \cdot \frac{1}{3} (x+1)^3 \Big|_1^3 = \pi \cdot \frac{1}{3} (64 - 8) = \underline{\underline{\frac{56\pi}{3} u^3}}$

$u = x+1$
 $u' = 1$

b) $f(x) = x^2$ $a=0$ $b=4$

$$V = \pi \int_0^4 (x^2)^2 dx = \pi \int_0^4 x^4 dx = \pi \cdot \frac{1}{5} x^5 \Big|_0^4 = \pi \left(\frac{1024}{5} - 0 \right) = \underline{\underline{\frac{1024\pi}{5} u^3}}$$

c) $f(x) = \frac{1}{x+1}$ $a=1$ $b=2$

$$V = \pi \int_1^2 \left(\frac{1}{x+1} \right)^2 dx = \pi \int_1^2 (x+1)^{-2} dx = \pi \frac{1}{-1} (x+1)^{-1} \Big|_1^2 = -\pi \cdot \frac{1}{x+1} \Big|_1^2$$

$u = x+1$
 $u' = 1$

$$= -\pi \left(\frac{1}{3} - \frac{1}{2} \right) = -\pi \cdot \left(-\frac{1}{6} \right) = \underline{\underline{\frac{\pi}{6} u^3}}$$

Ex 1.3.28

a) $f(x) = \sqrt{x}$ $g(x) = x^2$

pts d'∩ : $\sqrt{x} = x^2 \Leftrightarrow x = x^4 \Leftrightarrow x^4 - x = 0 \Leftrightarrow x(x^3 - 1) = 0$

$$\Leftrightarrow \underbrace{x}_{\substack{\downarrow \\ 0}} (x-1) \underbrace{(x^2+x+1)}_{\substack{\downarrow \\ 1}, \Delta < 0} = 0$$

$$V_1 = \pi \int_0^1 (\sqrt{x})^2 dx = \pi \int_0^1 x dx = \pi \cdot \left. \frac{1}{2} x^2 \right|_0^1 = \pi \left(\frac{1}{2} - 0 \right) = \frac{\pi}{2}$$

$$V_2 = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx = \pi \cdot \left. \frac{1}{5} x^5 \right|_0^1 = \pi \left(\frac{1}{5} - 0 \right) = \frac{\pi}{5}$$

$$V = |V_1 - V_2| = \left| \frac{\pi}{2} - \frac{\pi}{5} \right| = \pi \left| \frac{1}{2} - \frac{1}{5} \right| = \underline{\underline{\frac{3\pi}{10}}}$$

b) $f(x) = x^2 - 2x + 6$ $g(x) = -x^2 + 10$
 Sommets (1,5)

pts d'∩ : $x^2 - 2x + 6 = -x^2 + 10 \Leftrightarrow 2x^2 - 2x - 4 = 0$

$$\Leftrightarrow x^2 - x - 2 = 0 \Leftrightarrow (x-2)(x+1) = 0$$

$\downarrow$
2

$\downarrow$
-1

$$V_1 = \pi \int_{-1}^2 (x^2 - 2x + 6)^2 dx = \pi \int_{-1}^2 (x^4 + 4x^2 + 36 - 4x^3 + 12x^2 - 24x) dx$$

$$= \pi \int_{-1}^2 (x^4 - 4x^3 + 16x^2 - 24x + 36) dx = \pi \left(\frac{1}{5} x^5 - x^4 + \frac{16}{3} x^3 - 12x^2 + 36x \right) \Big|_{-1}^2$$

$$= \pi \left(\left(\frac{32}{5} - 16 + \frac{128}{3} - 48 + 72 \right) - \left(-\frac{1}{5} - 1 - \frac{16}{3} - 12 - 36 \right) \right) = \pi \left(\frac{856}{15} + \frac{818}{15} \right) = \frac{558\pi}{5}$$

$$V_2 = \pi \int_{-1}^2 (-x^2 + 10)^2 dx = \pi \int_{-1}^2 (x^4 - 20x^2 + 100) dx = \pi \left(\frac{1}{5} x^5 - \frac{20}{3} x^3 + 100x \right) \Big|_{-1}^2$$

$$= \pi \left(\left(\frac{32}{5} - \frac{160}{3} + 200 \right) - \left(-\frac{1}{5} + \frac{20}{3} - 100 \right) \right) = \pi \left(\frac{2296}{15} + \frac{1403}{15} \right) = \frac{1233\pi}{5}$$

$$\Rightarrow V = |V_1 - V_2| = \left| \frac{558\pi}{5} - \frac{1233\pi}{5} \right| = \pi \left| \frac{558}{5} - \frac{1233}{5} \right| = \underline{\underline{135\pi}}$$