

Règles de dérivation (suite)

$$3) (f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$4) \left(\frac{f}{g}\right)'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

en particulier :

$$\left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{g^2(x)}$$

dém 3) $(f \cdot g)'(a) = \lim_{x \rightarrow a} \frac{(f \cdot g)(x) - (f \cdot g)(a)}{x - a}$

$$= \lim_{x \rightarrow a} \frac{f(x)g(x) - f(a)g(a)}{x - a}$$

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 $f(a)g(x)$



$$= \lim_{x \rightarrow a} \frac{f(x)g(x) - \underline{f(a)g(x)} + \underline{f(a)g(x)} - f(a)g(a)}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{f(x)g(x) - f(a)g(x)}{x - a} + \lim_{x \rightarrow a} \frac{f(a)g(x) - f(a)g(a)}{x - a}$$

$$= \lim_{x \rightarrow a} g(x) \frac{f(x) - f(a)}{x - a} + \lim_{x \rightarrow a} f(a) \cdot \frac{g(x) - g(a)}{x - a}$$

$$= g(a) \cdot f'(a) + f(a) \cdot g'(a)$$

$$\Rightarrow (f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \quad \#$$

Exemples

$$a) f(x) = (3x^2 - 4)(1 - 5x)$$

$$u(x) = 3x^2 - 4$$

$$v(x) = 1 - 5x$$

$$u'(x) = 6x$$

$$v'(x) = -5$$

$$f'(x) = 6x(1 - 5x) + (-5)(3x^2 - 4)$$

$$= 6x - 30x^2 - 15x^2 + 20 = \underline{-45x^2 + 6x + 20}$$

$$b) f(x) = \frac{x^2 + 5}{3x - 1}$$

$$u = x^2 + 5$$

$$v = 3x - 1$$

$$u' = 2x$$

$$v' = 3$$

$$f'(x) = \frac{2x(3x - 1) - 3(x^2 + 5)}{(3x - 1)^2} = \frac{6x^2 - 2x - 3x^2 - 15}{(3x - 1)^2} = \frac{3x^2 - 2x - 15}{(3x - 1)^2}$$

$$\Delta = 4 + 180 = 184$$

ex 2.9.8 (et 9)

dém 4) (en exercice)

$$\left(\frac{f}{g}\right)'(a) = \lim_{x \rightarrow a} \frac{\frac{f(x)}{g(x)} - \frac{f(a)}{g(a)}}{x - a} = \lim_{x \rightarrow a} \frac{f(x)g(a) - f(a)g(x)}{g(x)g(a)} \cdot \frac{1}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{f(x)g(a) - f(a)g(x)}{x - a} \cdot \frac{1}{g(x)g(a)}$$



$$= \lim_{x \rightarrow a} \frac{f(x)g(a) - f(a)g(a) - f(a)g(x) + f(a)g(a)}{x - a} \cdot \lim_{x \rightarrow a} \frac{1}{g(x)g(a)}$$

$$= \lim_{x \rightarrow a} \left[g(a) \frac{f(x) - f(a)}{x - a} - f(a) \frac{g(x) - g(a)}{x - a} \right] \cdot \frac{1}{g^2(a)}$$

$$= \frac{g(a) f'(a) - f(a) \cdot g'(a)}{g^2(a)}$$

$$g^2(a)$$

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 $f(a)g(a)$