Systèmes d'équations

1. **Linéaires** du type $\begin{cases} 2x + 3y = -4 \\ 4x - y = 6 \end{cases}$ (2×2)

$\begin{cases} 2x + 3y = -4 \mid \cdot 2 \mid 1 \\ 4x - y = 6 \mid \cdot(-1) \mid 3 \end{cases}$

méthode des combinaisons linéaires

$\Rightarrow +\dfrac{\begin{matrix} 4x + 6y = -8 \\ -4x + y = -6 \end{matrix}}{\begin{matrix} 7y = -14 \\ y = -2 \end{matrix}}$

$\Rightarrow +\dfrac{\begin{matrix} 2x + 3y = -4 \\ 12x - 3y = 18 \end{matrix}}{\begin{matrix} 14x = 14 \\ x = 1 \end{matrix}}$

$$\Rightarrow S = \{(1\,;\,-2)\}$$

(3×3)

(1) $\begin{cases} 9x - 5y - 3z = 2 \\ -2x + 3y + z = 8 \\ 5x + 2y + 2z = 14 \end{cases}$ $\begin{vmatrix} 1 \\ 3 \\ \end{vmatrix}$ $\begin{vmatrix} \\ -2 \\ 1 \end{vmatrix}$
(2)
(3)

$+$ $\begin{array}{l} 9x - 5y - 3z = 2 \\ -6x + 9y + 3z = 24 \\ \hline 3x + 4y \qquad = 26 \end{array}$

$+$ $\begin{array}{l} 4x - 6y - 2z = -16 \\ 5x + 2y + 2z = 14 \\ \hline 9x - 4y \qquad = -2 \end{array}$

$\Rightarrow$ (4) $\begin{cases} 3x + 4y = 26 \\ 9x - 4y = -2 \end{cases}$ $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$
(5)

$\dfrac{}{12x \qquad = 24}$

$x \qquad = 2$

$\overset{(4)}{\Longrightarrow}$ $3 \cdot 2 + 4y = 26$

$4y = 20$

$y = 5$ $\overset{(2)}{\Longrightarrow}$ $-2 \cdot 2 + 3 \cdot 5 + z = 8$

$z = -3$

$\Rightarrow S = \{(2 ; 5 ; -3)\}$

Ex 2.5.21

n) $\begin{array}{l} (1) \\ (2) \\ (3) \end{array} \begin{cases} 2x + 3y + 4z = 47 \quad \Big| 1 \Big| 1 \\ 3x + 5y - 4z = 2 \quad \Big| 1 \Big| \\ 4x + 7y - 2z = 31 \quad \Big| \quad \Big| 2 \end{cases}$

(4) $5x + 8y = 49$

$2x + 3y + 4z = 47$

$\underline{8x + 14y - 4z = 62}$

$10x + 17y = 109 \quad (5)$

$\Rightarrow \begin{cases} 5x + 8y = 49 \quad \Big| 2 \\ 10x + 17y = 109 \quad \Big| -1 \end{cases}$

$\begin{array}{l} + \end{array} \begin{array}{l} 10x + 16y = 98 \\ \underline{-10x - 17y = -109} \\ \quad -y = -11 \\ \quad y = 11 \end{array}$

$\overset{(4)}{\Rightarrow} \quad 5x + 8 \cdot 11 = 49$

$5x = -39$

$x = -\dfrac{39}{5}$

$\overset{(3)}{\Rightarrow} \quad 4 \cdot \left(-\dfrac{39}{5}\right) + 7 \cdot 11 - 2z = 31$

$-2z = 31 - 77 + \dfrac{156}{5} = -\dfrac{74}{5}$

$z = -\dfrac{74}{5} \cdot \left(-\dfrac{1}{2}\right) = \dfrac{74}{10} = \dfrac{37}{5}$

$\Rightarrow \quad S = \left\{ \left(-\dfrac{39}{5}; 11; \dfrac{37}{5}\right)\right\}$