# Ex 2.9.15

a) $f(x) = 1 + 2x - x^3$ $\quad a = 1 \quad$ $t: \; y = mx + h$

1) dérivée : $f'(x) = -3x^2 + 2$

pente de la tgte : $m = f'(1) = -3 + 2 = -1 \;\Rightarrow\; t: \; y = -x + h$

2) $f(1) = 1 + 2 - 1 = 2 \;\Rightarrow\; T(1;2)$

$T(1;2) \in t \;\Rightarrow\; 2 = -1 + h \;\Leftrightarrow\; h = 3$

$\Rightarrow \quad t: \; y = -x + 3$

b) $f(x) = \dfrac{x+3}{x}$ $\quad a = 3 \quad$ $t: \; y = mx + h$

1) dérivée : $f'(x) = \dfrac{1 \cdot (x+3) - x \cdot 1}{x^2} = \dfrac{x + 3 - x}{x^2} = \dfrac{3}{x^2}$

pente de la tgte : $m = f'(3) = \dfrac{3}{9} = \dfrac{1}{3} \;\Rightarrow\; t: \; y = \dfrac{1}{3}x + h$

2) $f(3) = \dfrac{3+3}{3} = \dfrac{6}{3} = 2 \;\Rightarrow\; T(3;2)$

$T(3;2) \in t \;\Rightarrow\; 2 = \dfrac{1}{3} \cdot 3 + h \;\Leftrightarrow\; 2 = 1 + h \;\Leftrightarrow\; h = 1$

$\Rightarrow \quad t: \; y = \dfrac{1}{3}x + 1$

c) $f(x) = \sqrt{2x+1}$ , $a = 4$

1) dérivée : $f'(x) = \dfrac{2}{2\sqrt{2x+1}} = \dfrac{1}{\sqrt{2x+1}}$

pente de la tgte : $m = f'(4) = \dfrac{1}{\sqrt{8+1}} = \dfrac{1}{3} \;\Rightarrow\; t: \; y = \dfrac{1}{3}x + h$

2) $f(4) = \sqrt{8+1} = 3 \;\Rightarrow\; T(4;3)$

$T(4;3) \in t \;\Rightarrow\; 3 = \dfrac{1}{3} \cdot 4 + h \;\Leftrightarrow\; h = 3 - \dfrac{4}{3} = \dfrac{5}{3}$

$\Rightarrow \quad t: \; y = \dfrac{1}{3}x + \dfrac{5}{3}$

d) $f(x) = \dfrac{\sin(x)}{\sin(x)+\cos(x)}$ $\quad a = 0$

- $f'(x) = \dfrac{\cos(x)(\sin(x)+\cos(x)) - \sin(x)(\cos(x)-\sin(x))}{(\sin(x)+\cos(x))^2}$

$$= \frac{\cancel{\cos(x)\sin(x)} + \cos^2(x) - \cancel{\sin(x)\cos(x)} + \sin^2(x)}{(\sin(x)+\cos(x))^2}$$

$$= \frac{1}{(\sin(x)+\cos(x))^2}$$

$m = f'(0) = \dfrac{1}{1} = 1 \implies y = x + h$

- $f(0) = 0 \implies T(0;0) \in t$

$\implies t : y = x$

## Ex 2.9.16

$y = x^2$ et $m = -3$

$f'(x) = 2x \implies 2x = -3 \iff x = -\dfrac{3}{2}$

$f\left(-\dfrac{3}{2}\right) = \dfrac{9}{4} \implies P\left(-\dfrac{3}{2}; \dfrac{9}{4}\right)$

## Ex 2.9.17

$m = \dfrac{14-2}{1-(-3)} = 3 \implies f'(x) = 3x^2 - 2x - 5 = 3 \iff 3x^2 - 2x - 8 = 0$

$\Delta = 100 \implies x_{1,2} = \dfrac{2 \pm 10}{6} = \begin{cases} -4/3 \\ 2 \end{cases}$

## Ex 2.9.18

$m = 0 \implies f'(x) = \dfrac{1(x^2+9) - x \cdot 2x}{(x^2+9)^2} = \dfrac{-x^2+9}{(x^2+9)^2} = 0 \implies -x^2 + 9 = 0$

$\iff (-x+3)(x+3) = 0$

$\implies x = \begin{cases} -3 \implies f(-3) = -1/6 \implies P_1(-3; -1/6) \\ 3 \implies f(3) = 1/6 \implies P_2(3; 1/6) \end{cases}$

## Ex 2.9.21

$$f(x) = \frac{x^2 + ax + b}{cx^2 + dx - 2}$$

- Pas d'AH $\Rightarrow$ $c = 0$
- AV $x = 2$ $\Rightarrow$ $dx - 2 = x - 2 \Leftrightarrow d = 1 \Rightarrow f(x) = \frac{x^2 + ax + b}{x - 2}$
- $P(1; -2) \in y = f(x) \Rightarrow -2 = \frac{1 + a + b}{-1} \Leftrightarrow 2 = 1 + a + b \Leftrightarrow a + b = 1 \quad (1)$

$$f'(x) = \frac{(2x+a)(x-2) - 1(x^2 + ax + b)}{(x-2)^2}$$

$$\Rightarrow m = f'(1) = -5 \Rightarrow \frac{(2+a)\cdot(-1) - (1 + a + b)}{1} = -2a - b - 3 = -5$$

$$\Leftrightarrow -2a - b = -2 \quad (2)$$

(1) et (2) $\Rightarrow \begin{cases} a + b = 1 \\ -2a - b = -2 \end{cases} \left| \begin{matrix} 1 \\ 1 \end{matrix} \right. \Leftrightarrow \begin{cases} a + b = 1 \\ -a = -1 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = 0 \end{cases}$

## Ex 2.9.22

$f(x) = x^3 + ax^2 + bx \qquad T(-1; \ldots) \qquad t: y = x + 4$

$\Rightarrow m = f'(-1) = 1 \quad (1)$

$T(-1; \ldots) \in t \Rightarrow y = -1 + 4 = 3 \Rightarrow T(-1; 3) \Rightarrow f(-1) = 3 \quad (2)$

$f'(x) = 3x^2 + 2ax + b$

$(1) \Rightarrow 3 - 2a + b = 1 \Leftrightarrow$

$(2) \Rightarrow -1 + a - b = 3 \Leftrightarrow$

$\begin{cases} -2a + b = -2 \\ a - b = 4 \end{cases} \left| \begin{matrix} 1 \\ 1 \end{matrix} \right. \Leftrightarrow \begin{cases} -a = 2 \\ a - b = 4 \end{cases}$

$\Leftrightarrow \begin{cases} a = -2 \\ b = -6 \end{cases}$

## Ex 2.9.23

a) $f(x) = x^2$ et $P(5;9)$ $\quad P \notin y = f(x)$

avec formule

$f(a) = a^2$ et $f'(x) = 2x \Rightarrow f'(a) = 2a$

$\Rightarrow t: \; y = 2a(x-a) + a^2$

$P(5;9) \in t \Rightarrow 9 = 2a(5-a) + a^2 \Leftrightarrow 9 = 10a - 2a^2 + a^2$

$\Leftrightarrow a^2 - 10a + 9 = 0 \Leftrightarrow (a-9)(a-1) = 0 \Leftrightarrow a = \begin{cases} 9 \\ 1 \end{cases}$

Si $a = 9 \Rightarrow t_1: \; y = 18(x-9) + 81 \Leftrightarrow y = 18x - 81$

si $a = 1 \Rightarrow t_2: \; y = 2(x-1) + 1 \Leftrightarrow y = 2x - 1$

b) $f(x) = x^3$ et $P(0;-2)$ $\quad P \in y = f(x)$

avec formule

$f(a) = a^3$ et $f'(x) = 3x^2 \Rightarrow f'(a) = 3a^2$

$\Rightarrow t: \; y = 3a^2(x-a) + a^3$

$P(0;-2) \in t \Rightarrow -2 = 3a^2 \cdot (-a) + a^3 \Leftrightarrow -2 = -3a^3 + a^3 \Leftrightarrow 2a^3 = 2$

$\Leftrightarrow a^3 = 1 \Leftrightarrow a = 1$

$\Rightarrow t: \; y = 3(x-1) + 1 \Leftrightarrow y = 3x - 2$

## Ex 2.9.24

$y = x^3 + x^2$ et $P(0;0)$

$f(a) = a^3 + a^2$ et $f'(x) = 3x^2 + 2x \Rightarrow f'(a) = 3a^2 + 2a$

$t: \; y = (3a^2 + 2a)(x-a) + a^3 + a^2$

$P(0;0) \in t \Rightarrow 0 = (3a^2 + 2a)(-a) + a^3 + a^2 \Leftrightarrow -2a^3 - a^2 = 0$

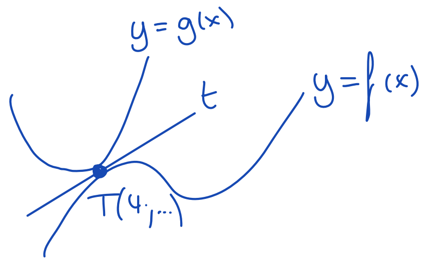
$\Leftrightarrow -a^2(2a+1) = 0 \Leftrightarrow a = \begin{cases} 0 \\ -1/2 \end{cases}$

si $a = 0 \Rightarrow f(0) = 0 \Rightarrow T_1(0;0)$ c'est le point P !

si $a = -\frac{1}{2} \Rightarrow f\left(-\frac{1}{2}\right) = -\frac{1}{8} + \frac{1}{4} = \frac{1}{8} \Rightarrow T_2\left(-\frac{1}{2}; \frac{1}{8}\right)$

## Ex 2.9.26

$y = \underbrace{x^3 + ax^2 + bx}_{= f(x)}$ et $y = \underbrace{x^2 - 6x}_{= g(x)}$




$f'(x) = 3x^2 + 2ax + b$ $\qquad$ $g'(x) = 2x - 6$

$f(4) = g(4) \Leftrightarrow 64 + 16a + 4b = 16 - 24 \Leftrightarrow 16a + 4b = -72$

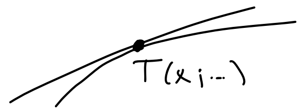
$\Leftrightarrow 4a + b = -18$

et $f'(4) = g'(4) \Leftrightarrow 48 + 8a + b = 8 - 6 \Leftrightarrow 8a + b = -46$

$\begin{cases} 4a + b = -18 & \mid \cdot (-1) \\ 8a + b = -46 & \mid +1 \end{cases} \Leftrightarrow \begin{cases} 4a = -28 \\ -28 + b = -18 \end{cases} \Leftrightarrow \begin{cases} a = -7 \\ b = 10 \end{cases}$

## Ex 2.9.27

$$y = \underbrace{\sqrt{x} + k}_{= f(x)} \quad \text{et} \quad y = \underbrace{\frac{x}{2} + 3}_{= g(x)}$$




$$\Rightarrow \quad f'(x) = \frac{1}{2\sqrt{x}} \quad \text{et} \quad g'(x) = \frac{1}{2}$$

$$f(x) = g(x) \iff \sqrt{x} + k = \frac{x}{2} + 3 \qquad (1)$$

$$\text{et} \quad f'(x) = g'(x) \iff \frac{1}{2\sqrt{x}} = \frac{1}{2} \iff \sqrt{x} = 1 \iff x = 1 \qquad (2)$$

(2) dans (1)

$$\Rightarrow \quad \sqrt{1} + k = \frac{1}{2} + 3 \iff k = \frac{5}{2} \quad \Rightarrow \quad f(1) \text{ ou } g(1) = \frac{1}{2} + 3 = \frac{7}{2}$$

$$\Rightarrow \quad T\left(1 \,;\, \frac{7}{2}\right)$$