Ex 4.2.1

$U = \{1; 2; 3; 4; 5; 6\}$ $\quad \# U = 6$

a) $P = \frac{1}{6}$ car $E = \{2\}$ $\quad \# E = 1$

b) $P = \frac{3}{6} = \frac{1}{2}$ car $E = \{2; 4; 6\}$ $\quad \# E = 3$

c) $P = \frac{2}{6} = \frac{1}{3}$ car $E = \{5; 6\}$ $\quad \# E = 2$

Ex 4.2.2

\# issues possibles : $36 = C_1^{36}$

a) $P = \frac{4}{36} = \frac{1}{9}$ car $E = \{A\heartsuit, A\diamondsuit, A\spadesuit, A\clubsuit\}$ $\quad \# E = 4$ ou C_1^4

b) $P = \frac{9}{36} = \frac{1}{4}$ car $E = \{A\diamondsuit, R\diamondsuit, \ldots, 6\diamondsuit\}$ $\quad \# E = 9$ ou C_1^9

c) $P = \frac{1}{36}$ car $E = \{V\heartsuit\}$ $\quad \# E = 1$ ou C_1^1

Ex 4.2.3

simultanément

\# issues possibles $= C_3^{36} = 7140$

a) $\frac{C_3^4}{C_3^{36}} = \frac{4}{7140} = \frac{1}{1785} \simeq 0{,}056\%$

b) $\frac{C_2^4 \cdot C_1^4}{C_3^{36}} = \frac{6 \cdot 4}{7140} = \frac{2}{595} \simeq 0{,}336\%$

successivement

\# issues possibles $= A_3^{36} = 36 \cdot 35 \cdot 34$

$\frac{A_3^4}{A_3^{36}} = \frac{4}{36} \cdot \frac{3}{35} \cdot \frac{2}{34} = \ldots$

place dame

$\frac{A_2^4 \cdot A_1^4 \cdot 3}{A_3^{36}} = \frac{4}{36} \cdot \frac{3}{35} \cdot \frac{4}{34} \cdot 3 = \ldots$

c) au moins 1 valet $\Leftrightarrow$ tout - aucun valet

$$1-\frac{C_3^{32}}{C_3^{36}} = 1-\frac{4960}{7140} = 1-\frac{248}{357} = \frac{109}{357} \cong 30{,}53\,\%$$

$$1-\frac{A_3^{32}}{A_3^{36}} = 1-\frac{32}{36}\cdot\frac{31}{35}\cdot\frac{30}{34} = \dots$$

Ex 4.2.4

$$\#\text{ issues possibles} = 2^4 = \underbrace{2}\cdot\underbrace{2}\cdot\underbrace{2}\cdot\underbrace{2} = \overline{A}_4^2 = 16$$

a) $P = \frac{1}{16}$ car # issue favorable $= 1 = \overset{P}{1}\cdot\overset{P}{1}\cdot\overset{F}{1}\cdot\overset{F}{1}$

b) $P = \frac{\overline{P}_4(2,2)}{2^4} = \frac{6}{16} = \frac{3}{8} = \frac{C_2^4}{2^4}$ $\begin{pmatrix} PPFF & PFFP \\ PFPF & FPPF \\ FPPF & FFPP \end{pmatrix}$

c) au plus 1 fois pile $\Leftrightarrow$ 0 fois pile ou 1 fois pile

$$1 + C_1^4 = 1+4 = 5$$

ou $1 + \overline{P}_4(3)$

$\Rightarrow P = \frac{5}{16}$

Ex 4.2.5

$$\#\text{ issues possibles} = 6^2 = 36$$

a) $P = \frac{6}{36} = \frac{1}{6}$ car $E = \{(1,1),(2,2),\dots,(6,6)\}$ $\#E = 6$

b) $P = \frac{2}{36} = \frac{1}{18}$ car $E = \{(2R,5B),(5R,2B)\}$ $\#E = 2$

c) $P = \frac{1}{36}$ car $E = \{(2R, 5B)\}$ $\# E = 1$

d) $E = \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\}$ $\# E = 6$

$\Rightarrow P = \frac{6}{36} = \frac{1}{6}$

e) $E = \{(1,1), (1,2), (2,1)\}$ $\# E = 3$ $\Rightarrow P = \frac{3}{36} = \frac{1}{12}$

f) somme au plus égale à 11 $\Leftrightarrow$ somme = 2 ou 3 ou 4 ou ... ou 11

$\Leftrightarrow$ tout − somme = 12

$\Leftrightarrow$ 36 − 1 = 35

$\Rightarrow P = \frac{35}{36}$

Ex 4.2.6

simultanément

$$\frac{C_3^4 \cdot C_{10}^{48}}{C_{13}^{52}} \cong 4{,}12\%$$

successivement

nbre de place pour les rois.

$$\frac{A_3^4 \cdot A_{10}^{48} \cdot C_3^{13}}{A_{13}^{52}} \cong 4{,}12\%$$

Ex 4.2.8

$\#$ issues possibles $= C_3^{36} = 7140$

1 couleur

3 cartes par couleur

a) $\frac{C_1^4 \cdot C_3^9}{C_3^{36}} = \frac{336}{7140} = \frac{4}{85} \cong 4{,}71\%$

b) $\frac{C_3^4}{C_3^{36}} = \frac{4}{7140} = \frac{1}{1785} \cong 0{,}06\%$

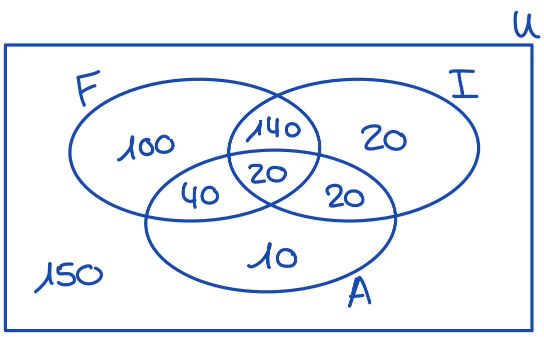
c) $\frac{C_1^4 \cdot C_2^4}{C_3^{36}} = \frac{4 \cdot 6}{7140} = \frac{2}{595} \cong 0{,}34\%$

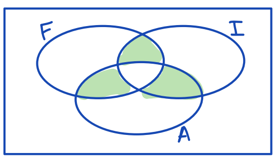
d) $\dfrac{\overset{\text{1 couleur}}{C_1^4}\cdot \overset{\text{2 cartes}}{C_2^9}\cdot \overset{\text{1 carte autre couleur}}{C_1^{27}}}{C_3^{36}} = \dfrac{4\cdot 36\cdot 27}{7140} = \dfrac{324}{595} \cong 54,45\,\%$

e) $\dfrac{C_2^{18}\cdot C_1^{18}}{C_3^{36}} = \dfrac{2754}{7140} = \dfrac{27}{70} \cong 38,57\,\%$

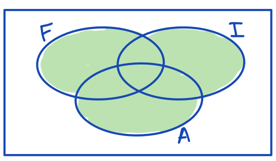
f) $\dfrac{C_1^4\cdot C_1^4\cdot C_1^4}{C_3^{36}} = \dfrac{4\cdot 4\cdot 4}{7140} = \dfrac{16}{1785} \cong 0,896\,\%$

Ex 4.2.9



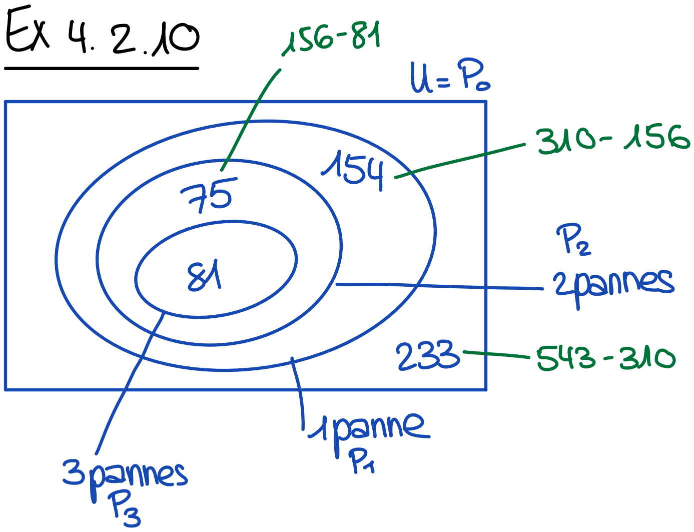
a) 

$P = \dfrac{140+40+20}{500} = \dfrac{200}{500} = \dfrac{2}{5} = 40\,\%$

b) 

$P = \dfrac{300+20+20+10}{500} = \dfrac{350}{500} = \dfrac{7}{10} = 70\,\%$

Ex 4.2.10



a) $P = \dfrac{154}{543} \cong 28,36\,\%$

b) $P = \dfrac{154+233}{543} = \dfrac{387}{543} \cong 71,27\,\%$

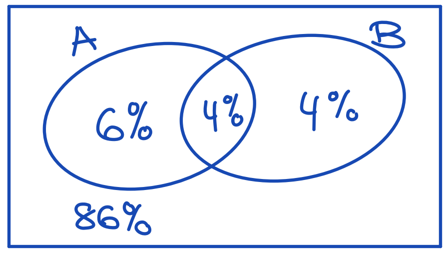
Ex 4.2.14

a) $P(A \cup B) = 6\% + 4\% + 4\% = \underline{14\%}$

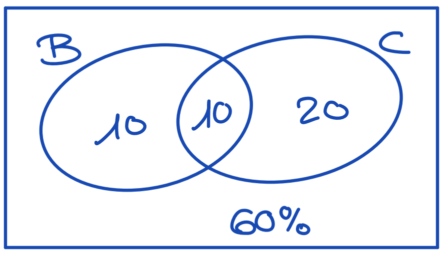
b) $P = \underline{6\%}$

c) $P = 6 + 4 = \underline{10\%}$

d) $P = 100\% - 6\% - 4\% - 4\% = \underline{86\%}$



Ex 4.2.15



$P(\overline{B \cup C}) = 60\% \Rightarrow P(B \cup C) = 100\% - 60\% = \underline{40\%}$

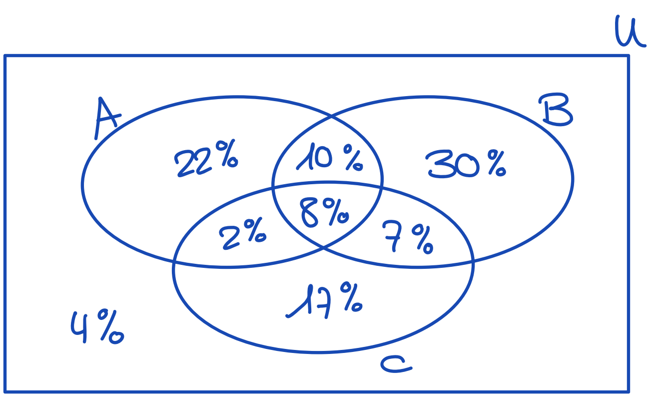
$P(B) = 20\%$ et $P(C) = 30\%$

Comme $P(B \cup C) = P(B) + P(C) - P(B \cap C) \Leftrightarrow 40 = 20 + 30 - P(B \cap C)$

$\Leftrightarrow 40 = 50 - P(B \cap C)$

$\Rightarrow P(B \cap C) = \underline{10}\%$

Ex 4.2.16



a) $P(A \cup B \cup C) = 42\% + 30\% + 7\% + 17\% = \underline{96\%}$

b) $P(\overline{A \cup B \cup C}) = 100\% - 96\% = \underline{4\%}$

c) $P = 10\% + 2\% + 7\% = \underline{19\%}$

d) $P = \underline{22\%}$

Ex 4.3.1

$\#A = 5 \quad \#B = 30 \quad \#C\ 18$

$$P(A) = \underline{\frac{5}{36}}$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{4/36}{30/36} = \frac{4}{30} = \underline{\frac{2}{15}}$$

$$P(A|C) = \frac{P(A\cap C)}{P(C)} = \frac{2/36}{18/36} = \frac{2}{18} = \underline{\frac{1}{9}}$$

$$P(A|\bar{B}) = \frac{P(A\cap \bar{B})}{P(\bar{B})} = \frac{1/36}{6/36} = \underline{\frac{1}{6}}$$

$$P(A|\bar{C}) = \frac{P(A\cap \bar{C})}{P(\bar{C})} = \frac{3/36}{18/36} = \frac{3}{18} = \underline{\frac{1}{6}}$$

Ex 4.3.2

$\#A = 9 \quad \#B = 1 \quad \#C = 12$

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{1/36}{9/36} = \underline{\frac{1}{9}}$$

$$P(A|C) = \frac{P(A\cap C)}{P(C)} = \frac{9/36}{12/36} = \frac{9}{12} = \underline{\frac{3}{4}}$$

$$P(B|C) = \frac{P(B\cap C)}{P(C)} = \frac{1/36}{12/36} = \underline{\frac{1}{12}}$$

$$P(C|B) = \frac{P(C\cap B)}{P(B)} = \frac{1/36}{1/36} = \underline{1}$$

Ex 4.3.7

a) $p = \dfrac{C_2^4}{C_2^8} = \dfrac{6}{28} = \dfrac{3}{14} \cong 0{,}2143 = 21{,}43\,\%$

b) $p = \dfrac{C_1^1}{C_2^8} = \dfrac{1}{28} \cong 0{,}0357 = 3{,}57\,\%$

c) $p =$ "tout- aucun" $= 1 - \dfrac{C_2^4}{C_2^8} = 1 - \dfrac{3}{14} = \dfrac{11}{14} \cong 0{,}7857$
$= 78{,}57\%$

d) i) $p = \dfrac{\overset{a)}{3/14}}{\underset{c)}{11/14}} = \dfrac{3}{11}$

ii) $p = \dfrac{\overset{b)}{\dfrac{1}{28}} + \overset{1AR\,et\,1AN}{\dfrac{C_1^2 C_1^2}{28}}}{1 - \dfrac{C_2^6}{28}} = \dfrac{5/28}{1 - 15/28} = \dfrac{5/28}{13/28} = \dfrac{5}{13}$

iii) $p = \dfrac{\dfrac{C_1^1 \cdot C_1^3}{28}}{1 - \dfrac{C_2^7}{28}} = \dfrac{3/28}{1 - \dfrac{21}{28}} = \dfrac{3/28}{7/28} = \dfrac{3}{7}$

Ex 4.3.8

A : somme obtenue sup. à 9 $\qquad \#A = 6$

B : 1e dé a donné 5 $\qquad \#B = 6$

C : au moins 1 dé a donné 5 $\qquad \#C = 11$

a) $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{2/36}{6/36} = \dfrac{2}{6} = \dfrac{1}{3}$

b) $P(A|C) = \dfrac{P(A\cap C)}{P(C)} = \dfrac{3/36}{11/36} = \dfrac{3}{11}$

D : "deux chiffres différents" $\qquad \#D = 30$

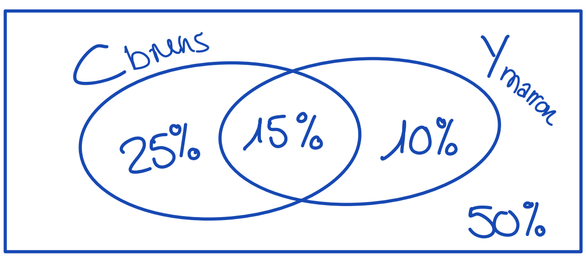
E : "$\Sigma = 6$" $\qquad \#E = 5$

F : "$\Sigma < 5$ $\qquad \#F = 6$

c) $P(E|D) = \dfrac{P(E\cap D)}{P(D)} = \dfrac{4}{30} = \dfrac{2}{15}$

d) $P(F|D) = \dfrac{P(F\cap D)}{P(D)} = \dfrac{4}{30} = \dfrac{2}{15}$

Ex 4.3.9



C : "cheveux bruns" $\qquad P(C) = 40\%$

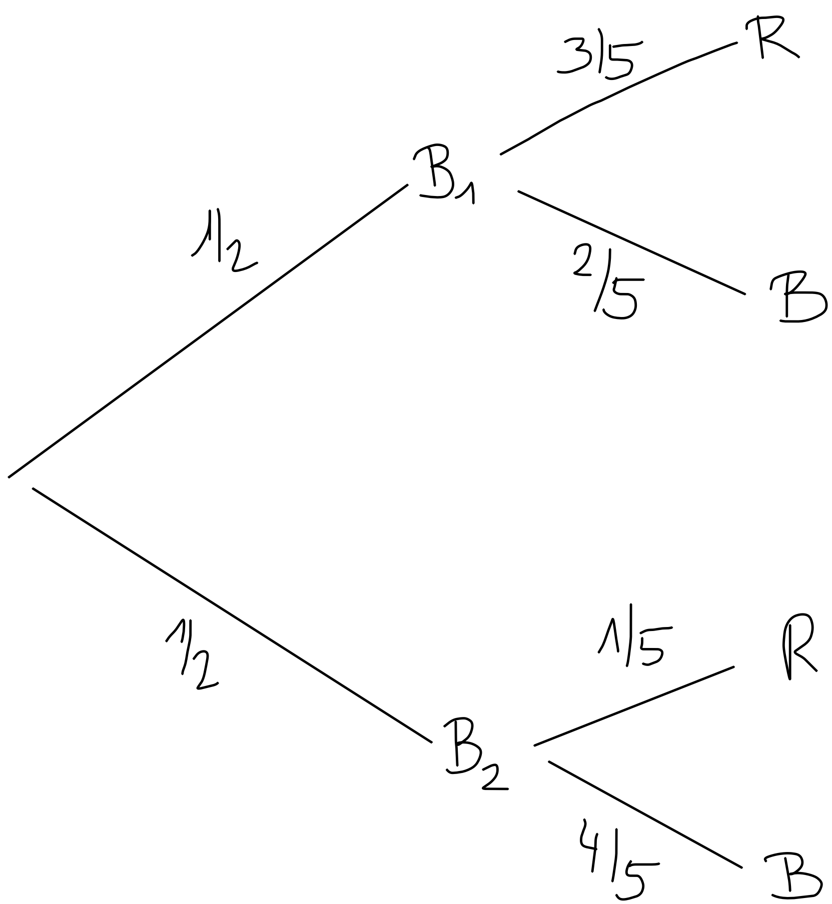
Y : "yeux marron" $\qquad P(Y) = 25\%$

a) $P(Y|C) = \dfrac{P(Y\cap C)}{P(C)} = \dfrac{15\%}{40\%} = \dfrac{15}{40} = \dfrac{3}{8} = 0{,}375 = 37{,}5\ \%$

b) $P(\overline{C}|Y) = \dfrac{P(\overline{C}\cap Y)}{P(Y)} = \dfrac{10}{25} = \dfrac{2}{5} = 0{,}4 = 40\%$

c) $P(\overline{C}\cap\overline{Y}) = 100\% - 25\% - 15\% - 10\% = 50\%$

4.3.11 On lance une pièce de monnaie bien équilibrée. Si l'on obtient face, on tire une bille d'une boîte B_1 contenant 3 billes rouges et 2 bleues. Sinon, on tire une bille d'une boîte B_2 contenant 2 billes rouges et 8 bleues. Sachant qu'on a tiré une bille rouge, quelle est la probabilité qu'elle provienne de la boîte B_1 ?



$$\frac{1}{2}\cdot\frac{3}{5} = P(B_1 \cap R) = \frac{3}{10}$$

$$\frac{1}{2}\cdot\frac{1}{5} = \frac{1}{10}$$

$$P(B_1 | R) = \frac{P(B_1 \cap R)}{P(R)} = \frac{3/10}{3/10 + 1/10} = \frac{3/10}{4/10} = \frac{3}{4}$$

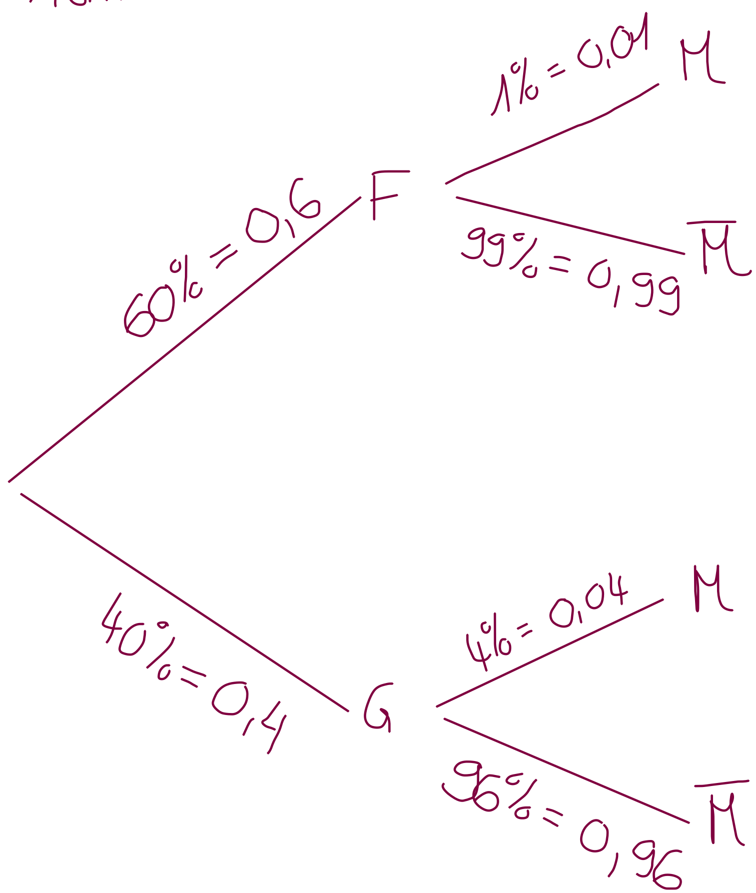
4.3.12 Dans un gymnase, 4% des garçons et 1% des filles mesurent plus de 1,8 m. Or, 60% des élèves sont des filles. On choisit un élève au hasard et on constate qu'il mesure plus de 1,8 m. Quelle est la probabilité que ce soit une fille ?

M : plus de 1,8m

$$\frac{60}{100} \cdot \frac{1}{100} = \frac{60}{10'000}$$

$60\% \cdot 1\% \neq 60\%$

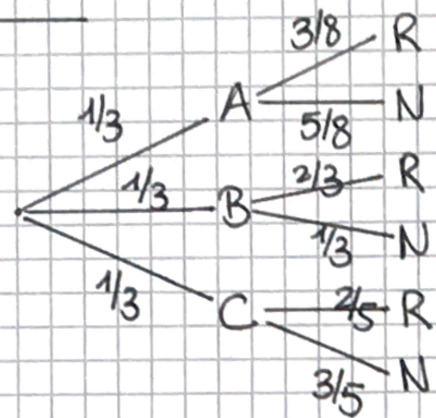
$0,6 \cdot 0,01 = \underline{0,006} = 0,6\%$



$0,4 \cdot 0,04 = 0,016 = 1,6\%$

$$P(F \mid M) = \frac{P(F \cap M)}{P(M)} = \frac{0,006}{0,006 + 0,016} = 0,\overline{27} = 27,\overline{27}\% = \frac{3}{11}$$

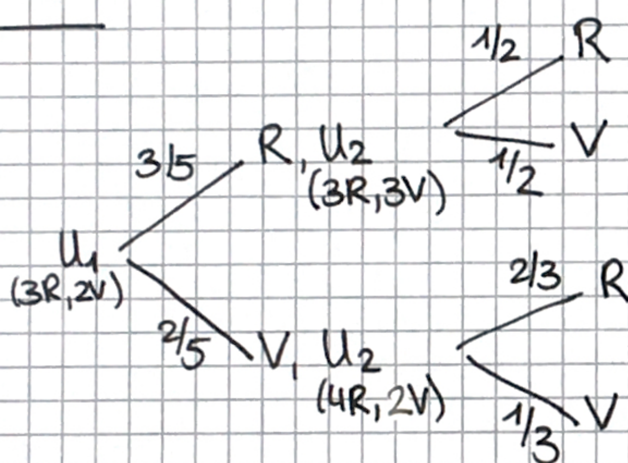
Ex 4.3.14



a) $P(R) = \frac{1}{3}\cdot\frac{3}{8} + \frac{1}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{2}{5} = \frac{1}{8} + \frac{2}{9} + \frac{2}{15} = \frac{173}{360} \cong 48{,}05\%$

b) $P(A|R) = \frac{P(A\cap R)}{P(R)} = \frac{1/3 \cdot 3/8}{173/360} = \frac{45}{173} \cong 26{,}01\,\%$

Ex 4.3.15

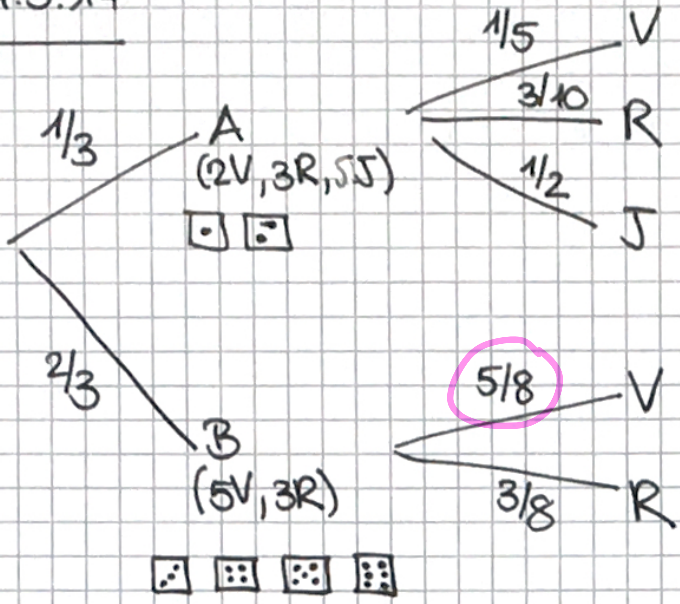


a) $P(\_,R) = \frac{3}{5}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{2}{3} = \frac{3}{10} + \frac{4}{15} = \frac{17}{30} = 56{,}6\,\%$

b) $P(\_,R|R,\_) = \frac{P(R,R)}{P(R,\_)} = \frac{3/5 \cdot 1/2}{3/5} = \frac{1}{2} = 50\%$

c) $P(R,\_|\_,R) = \frac{P(R,R)}{P(\_,R)} = \frac{3/5 \cdot 1/2}{17/30} = \frac{3/10}{17/30} = \frac{9}{17} \cong 52{,}94\%$

Ex 4.3.17



a) $P(V) = \frac{1}{3}\cdot\frac{1}{5} + \frac{2}{3}\cdot\frac{5}{8} = \frac{1}{15} + \frac{5}{12} = \frac{29}{60} = 48,\overline{3}\,\%$

b) $P(V|B) = \frac{P(V\cap B)}{P(B)} = \frac{\frac{2}{3}\cdot\frac{5}{8}}{\frac{2}{3}} = \frac{5}{8} = 62,5\%$

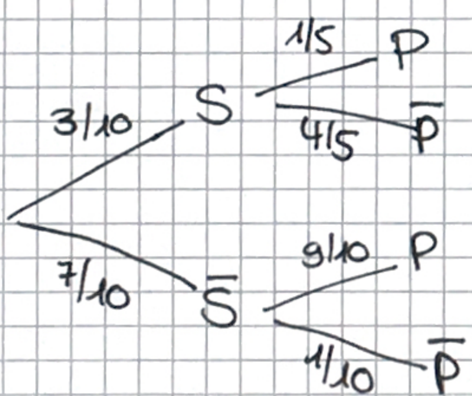
c) $P(A|R) = \frac{P(A\cap R)}{P(R)} = \frac{\frac{1}{3}\cdot\frac{3}{10}}{\frac{1}{3}\cdot\frac{3}{10}+\frac{2}{3}\cdot\frac{3}{8}} = \frac{\frac{1}{10}}{\frac{7}{20}} = \frac{2}{7} \cong 28,57\%$

d) $P(B|J) = 0$

Ex 4.3.18

P : être parmi les 10 premiers

S : course ensoleillée



$P(\bar{S}|P) = \frac{P(\bar{S}\cap P)}{P(P)} = \frac{\frac{7}{10}\cdot\frac{9}{10}}{\frac{3}{10}\cdot\frac{1}{5}+\frac{7}{10}\cdot\frac{9}{10}} = \frac{21}{23} \cong 91,3\,\%$