Ex 2.10.2

a) $f(x) = x^3 - 3x \qquad ED(f) = \mathbb{R}$

$f'(x) = 3x^2 - 3 = 3(x^2-1) = 3(x+1)(x-1)$

$-1 \qquad 1$ zéros de f'

x		-1		1	
f'	$+$	0	$-$	0	$+$
f	$\nearrow$	Max	$\searrow$	min	$\nearrow$

$f(-1) = (-1)^3 - 3\cdot(-1) = -1+3 = 2 \implies \text{Max}(-1;2)$

$f(1) = 1^3 - 3\cdot 1 = 1-3 = -2 \implies \text{min}(1;-2)$

b) $f(x) = -x^4 + 2x^2 + 12 \qquad ED(f) = \mathbb{R}$

$f'(x) = -4x^3 + 4x = -4x(x^2-1) = -4x(x+1)(x-1)$

$0 \quad -1 \quad 1$ zéros de f'

x		-1		0		1	
sgn f'	$+$	0	$-$	0	$+$	0	$-$
f	$\nearrow$	Max_1	$\searrow$	min	$\nearrow$	Max_2	$\searrow$

$f(-1) = 13 \implies \text{Max}_1(-1;13)$

$f(0) = 12 \implies \text{min}(0;12)$

$f(1) = 13 \implies \text{Max}_2(1;13)$

c) $f(x) = (x+2)^3(x-3)^2$ $\qquad$ $ED(f) = \mathbb{R}$

$u = (x+2)^3$
$u' = 3(x+2)^2\cdot 1$
$= 3(x+2)^2$

$v = (x-3)^2$
$v' = 2(x-3)\cdot 1$
$= 2(x-3)$

$$f'(x) = 3(x+2)^2(x-3)^2 + (x+2)^3\cdot 2(x-3)$$

$$= 3(x+2)^2(x-3)^2 + 2(x+2)^3(x-3)$$

$$= (x+2)^2(x-3)\left[\underbrace{3(x-3)+2(x+2)}_{3x-9+2x+4}\right]$$

$$= (x+2)^2(x-3)(\underbrace{5x-5}_{5(x-1)}) = 5(x+2)^2(x-3)(x-1)$$

$-2\ (2)$ $\quad 3 \quad 1$ $\quad$ zéros de f'

x		-2		1		3	
sgn f'	$+$	0 (2)	$+$	0	$-$	0	$+$
f	↗	palier	↗	Max	↘	min	↗

$f'(1000): +$

palier : $f(-2) = (-2+2)^3(-2-3)^2 = 0\cdot(-5)^2 = 0 \Rightarrow$ palier $(-2;0)$

Max : $f(1) = (1+2)^3(1-3)^2 = 3^3\cdot(-2)^2 = 27\cdot 4 = 108 \Rightarrow$ Max $(1;108)$

min : $f(3) = (3+2)^3(3-3)^2 = 5^3\cdot 0 = 0 \Rightarrow$ min $(3;0)$

d) $f(x) = \dfrac{2x-3}{x+5}$ $\qquad$ $ED(f) = \mathbb{R} - \{-5\}$

$$f'(x) = \frac{2(x+5) - (2x-3)\cdot 1}{(x+5)^2} = \frac{2x+10-2x+3}{(x+5)^2} = \frac{13}{(x+5)^2}$$

pas de zéro

v.i. : -5 (2)

x		5	
f'	$+$	‖ (2)	$+$
f	↗		↗

pas d'extremum

e) $f(x) = \dfrac{(x-1)^2}{x+2}$ $\qquad ED(f) = \mathbb{R} - \{-2\}$

$$f'(x) = \frac{2(x-1)(x+2) - (x-1)^2 \cdot 1}{(x+2)^2} = \frac{(x-1)\left[2(x+2) - (x-1)\right]}{(x+2)^2} = \frac{(x-1)(x+5)}{(x+2)^2}$$

x		-5		-2		1	
f'	$+$	0	$-$	$\Vert$	$-$	0	$+$
f	$\nearrow$	Max	$\searrow$	$\Vert$ (2)	$\searrow$	min	$\nearrow$

$f(-5) = \dfrac{36}{-3} = -12 \implies \text{Max}\,(-5;-12)$

$f(1) = 0 \implies \text{min}\,(1;0)$

f) $f(x) = \dfrac{x}{x^2+1}$ $\qquad ED(f) = \mathbb{R}$

$$f'(x) = \frac{1\cdot(x^2+1) - x\cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{(1+x)(1-x)}{(x^2+1)^2}$$

zéros : ± 1

x		-1	$+$	1	
f'	$-$	0	$+$	0	$-$
f	$\searrow$	min	$\nearrow$	Max	$\searrow$

$f(-1) = -\dfrac{1}{2} \implies \text{min}\left(-1;-\dfrac{1}{2}\right)$

$f(1) = \dfrac{1}{2} \implies \text{Max}\left(1;\dfrac{1}{2}\right)$

g) $f(x) = x^2\sqrt{6-x^2}$ $\qquad$ $ED(f) = [-\sqrt{6}\,;\sqrt{6}]$

$$f'(x) = 2x\sqrt{6-x^2} + x^2 \cdot \frac{-2x}{2\sqrt{6-x^2}} = 2x\sqrt{6-x^2} + \frac{-x^3}{\sqrt{6-x^2}} = \frac{2x(6-x^2)-x^3}{\sqrt{6-x^2}}$$

$$= \frac{-3x^3+12x}{\sqrt{6-x^2}} = \frac{-3x(x^2-4)}{\sqrt{6-x^2}} = \frac{-3x(x+2)(x-2)}{\sqrt{6-x^2}}$$

zéros de f' : 0 et ± 2 $\qquad$ $ED(f') =]-\sqrt{6}\,;\sqrt{6}[$ ou pôles : $\pm\sqrt{6}$

x	$-\sqrt{6}$		-2		0		2		$\sqrt{6}$
sgn(f')	‖	+	0	−	0	+	0	−	‖
croiss(f)	min	↗	Max	↘	min	↗	Max	↘	min

min $(-\sqrt{6}\,;0)$; Max $(-2\,;4\sqrt{2})$, min $(0\,;0)$; Max $(2\,;4\sqrt{2})$ min $(\sqrt{6}\,;0)$

h) $f(x) = \sin(x)(1+\cos(x))$ $\qquad$ $ED(f) = [0\,;2\pi]$

$$f'(x) = \cos(x)(1+\cos(x)) + \sin(x)\cdot(-\sin(x)) = \cos(x) + \cos^2(x) \underbrace{- \sin^2(x)}_{=\cos^2(x)-1}$$

$$= 2\cos^2(x) + \cos(x) - 1 = (2\cos(x)-1)(\cos(x)+1)$$

$ED(f') = [0\,;2\pi]$

zéros de f' : $\cos(x) = \frac{1}{2} \Leftrightarrow x = \pm\frac{\pi}{3} + k\cdot 2\pi \Rightarrow x_1 = \frac{\pi}{3}\,;\ x_2 = \frac{5\pi}{3}$

$\cos(x) = -1 \Leftrightarrow x = \pm\pi + k\cdot 2\pi \Rightarrow x_3 = \pi$

x	0		$\pi/3$		π		$5\pi/3$		2π
f'		+	0	−	0	−	0	+	
f		↗	Max	↘	palier	↘	min	↗	

Max $\left(\frac{\pi}{3}\,;\frac{3\sqrt{3}}{4}\right)$ $\quad$ palier $(\pi\,;0)$ $\quad$ min $\left(\frac{5\pi}{3}\,;-\frac{3\sqrt{3}}{4}\right)$