

Ex 2.10.2

a) $f(x) = x^3 - 3x$ $ED(f) = \mathbb{R}$

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$$

\downarrow \downarrow
-1 1 zeros de f' .

x	-1	1
f'	+	0
f	Max	min

$$f(-1) = (-1)^3 - 3 \cdot (-1) = -1 + 3 = 2 \Rightarrow \underline{\text{Max}(-1; 2)}$$

$$f(1) = 1^3 - 3 \cdot 1 = 1 - 3 = -2 \Rightarrow \underline{\text{min}(1; -2)}$$

b) $f(x) = -x^4 + 2x^2 + 12$ $ED(f) = \mathbb{R}$

$$f'(x) = -4x^3 + 4x = -4x(x^2 - 1) = -4x(x+1)(x-1)$$

\downarrow \downarrow \downarrow
0 -1 1 zeros de f'

x	-1	0	1
f'	+	0	-
f	Max ₁	min	Max ₂

$$f(-1) = 13 \Rightarrow \text{Max}_1(-1; 13)$$

$$f(0) = 12 \Rightarrow \text{min}(0; 12)$$

$$f(1) = 13 \Rightarrow \text{Max}_2(1; 13)$$

c) $f(x) = (x+2)^3(x-3)^2$ $ED(f) = \mathbb{R}$

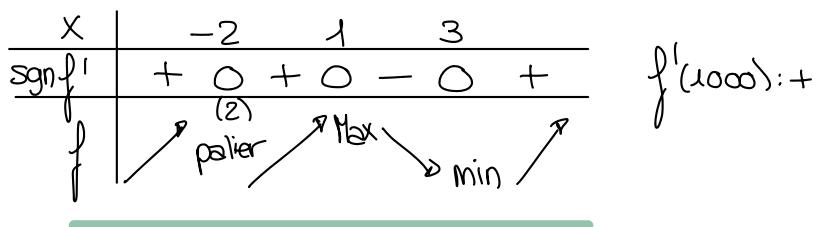
$$\begin{aligned}f'(x) &= 3(x+2)^2(x-3)^2 + (x+2)^3 \cdot 2(x-3) \\&= 3\underline{(x+2)^2}\underline{(x-3)^2} + 2\underline{(x+2)^3}\underline{(x-3)} \\&= (x+2)^2(x-3) \left[\underbrace{3(x-3) + 2(x+2)}_{3x-9+2x+4} \right]\end{aligned}$$

$$\begin{aligned}u &= (x+2)^3 \\u' &= 3(x+2)^2 \cdot 1 \\&= 3(x+2)^2\end{aligned}\quad \begin{aligned}v &= (x-3)^2 \\v' &= 2(x-3) \cdot -1 \\&= 2(x-3)\end{aligned}$$

$$= (x+2)^2(x-3)(\underbrace{5x-5}_{5(x-1)}) = 5(x+2)^2(x-3)(x-1)$$

\downarrow \downarrow \downarrow

zéros de f'



platier : $f(-2) = (-2+2)^3(-2-3)^2 = 0 \cdot (-5)^2 = 0 \Rightarrow \underline{\text{platier } (-2; 0)}$

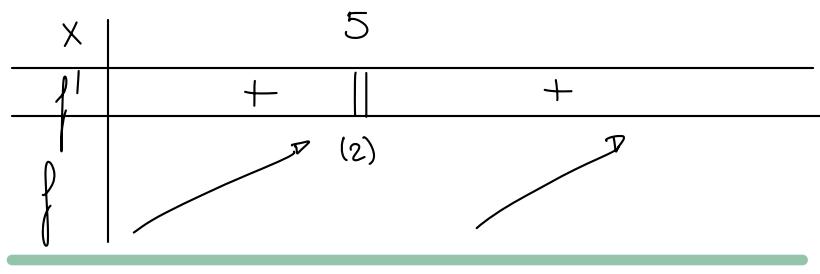
Max : $f(1) = (1+2)^3(1-3)^2 = 3^3 \cdot (-2)^2 = 27 \cdot 4 = 108 \Rightarrow \underline{\text{Max}(1; 108)}$

min : $f(3) = (3+2)^3(3-3)^2 = 5^3 \cdot 0 = 0 \Rightarrow \underline{\text{min}(3; 0)}$

d) $f(x) = \frac{2x-3}{x+5}$ $ED(f) = \mathbb{R} - \{-5\}$

$$f'(x) = \frac{2(x+5) - (2x-3) \cdot 1}{(x+5)^2} = \frac{2x+10-2x+3}{(x+5)^2} = \frac{13}{(x+5)^2}$$

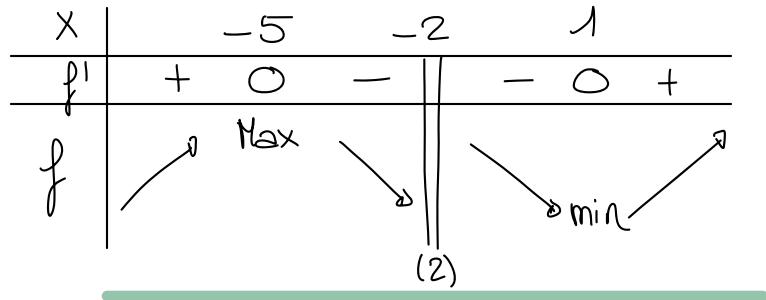
pas de zéro
v.i. : -5 (2)



pas d'extremum

$$e) f(x) = \frac{(x-1)^2}{x+2} \quad ED(f) = \mathbb{R} - \{-2\}$$

$$f'(x) = \frac{2(x-1)(x+2) - (x-1)^2 \cdot 1}{(x+2)^2} = \frac{(x-1)[2(x+2) - (x-1)]}{(x+2)^2} = \frac{(x-1)(x+5)}{(x+2)^2}$$

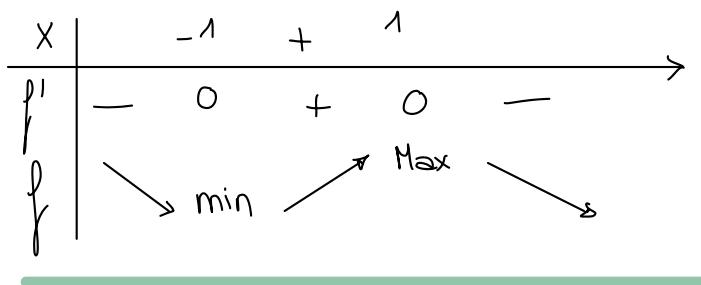


$$f(-5) = \frac{36}{-3} = -12 \Rightarrow \underline{\text{Max } (-5; -12)}$$

$$f(1) = 0 \Rightarrow \underline{\text{min } (1; 0)}$$

$$f(x) = \frac{x}{x^2+1} \quad ED(f) = \mathbb{R}$$

$$f'(x) = \frac{1 \cdot (x^2+1) - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{(1+x)(1-x)}{(x^2+1)^2} \quad \text{zeros: } \pm 1$$



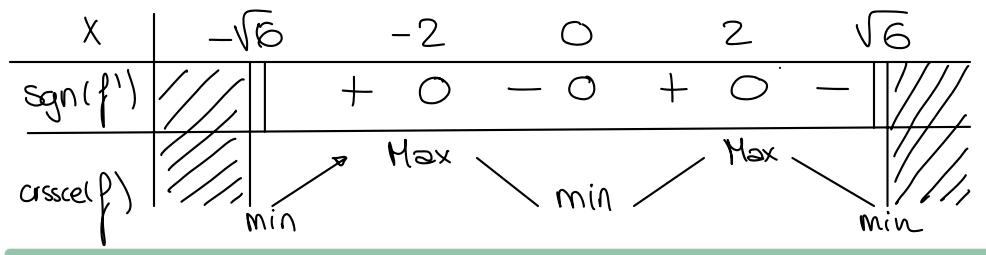
$$f(-1) = -\frac{1}{2} \Rightarrow \underline{\text{min } (-1; -\frac{1}{2})}$$

$$f(1) = \frac{1}{2} \Rightarrow \underline{\text{Max } (1; \frac{1}{2})}$$

$$g) f(x) = x^2 \sqrt{6-x^2} \quad ED(f) = [-\sqrt{6}; \sqrt{6}]$$

$$\begin{aligned} f'(x) &= 2x\sqrt{6-x^2} + x^2 \cdot \frac{-2x}{2\sqrt{6-x^2}} = 2x\sqrt{6-x^2} + \frac{-x^3}{\sqrt{6-x^2}} = \frac{2x(6-x^2) - x^3}{\sqrt{6-x^2}} \\ &= \frac{-3x^3 + 12x}{\sqrt{6-x^2}} = \frac{-3x(x^2-4)}{\sqrt{6-x^2}} = \frac{-3x(x+2)(x-2)}{\sqrt{6-x^2}} \end{aligned}$$

zéros de f' : 0 et ± 2 $ED(f') = [-\sqrt{6}; \sqrt{6}]$ ou pôles: $\pm\sqrt{6}$



$$\underline{\min(-\sqrt{6}; 0)} \quad ; \quad \underline{\max(-2; 4\sqrt{2})}, \underline{\min(0; 0)} \quad ; \quad \underline{\max(2; 4\sqrt{2})} \quad \underline{\min(\sqrt{6}; 0)}$$

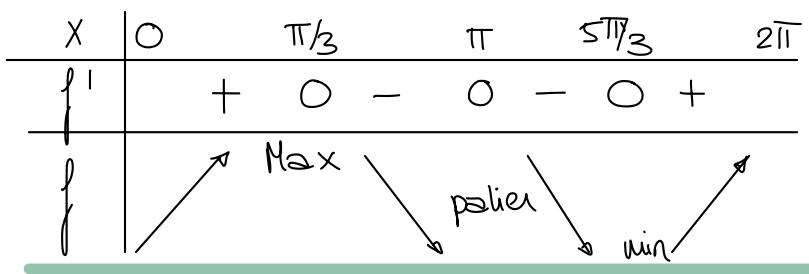
$$h) f(x) = \sin(x)(1+\cos(x)) \quad ED(f) = [0; 2\pi]$$

$$\begin{aligned} f'(x) &= \cos(x)(1+\cos(x)) + \sin(x) \cdot (-\sin(x)) = \cos(x) + \cos^2(x) - \underbrace{\sin^2(x)}_{=\cos^2(x)-1} \\ &= 2\cos^2(x) + \cos(x) - 1 = (2\cos(x) - 1)(\cos(x) + 1) \end{aligned}$$

$$ED(f') = [0; 2\pi]$$

zéros de f' : $\cos(x) = \frac{1}{2} \Leftrightarrow x = \pm \frac{\pi}{3} + k \cdot 2\pi \Rightarrow x_1 = \frac{\pi}{3}; x_2 = \frac{5\pi}{3}$

$$\cos(x) = -1 \Leftrightarrow x = \pm \pi + k \cdot 2\pi \Rightarrow x_3 = \pi$$



$$\underline{\max(\frac{\pi}{3}; \frac{3\sqrt{3}}{4})} \quad \underline{\text{polier}(\pi; 0)} \quad \underline{\min(\frac{5\pi}{3}; -\frac{3\sqrt{3}}{4})}$$