

Puissances à exposant entier

Ex 1.1.1

$$a) 2^4 \cdot 3^4 = (2 \cdot 3)^4 = \underline{6^4}$$

$$b) 2^3 \cdot (-3)^3 \cdot 4^3 = (2 \cdot (-3) \cdot 4)^3 = \underline{-24^3} \quad (= (-24)^3)$$

$$c) 3^6 \cdot 5^6 = (3 \cdot 5)^6 = \underline{15^6}$$

$$d) 5^0 \cdot 5^1 \cdot 5^2 \cdot \dots \cdot 5^{10} = 5^{0+1+2+\dots+10} = \underline{5^{55}}$$

$$e) 3^2 \cdot 5^2 \cdot 15^3 = (3 \cdot 5)^2 \cdot 15^3 = 15^2 \cdot 15^3 = 15^{2+3} = \underline{15^5}$$

$$f) \frac{5^8}{5^6} = 5^{8-6} = \underline{5^2}$$

$$g) \frac{5^6}{5^8} = \underline{\frac{1}{5^2}}$$

$$h) \left(-\frac{2}{3}\right)^5 = \underline{-\frac{2^5}{3^5}}$$

$$i) \frac{7 \cdot 7^5 \cdot 7^0 \cdot 7}{7^3 \cdot 7^4} = \frac{7^{1+5+0+1}}{7^{3+4}} = \frac{7^7}{7^7} = \underline{1}$$

Ex 1.1.2

$$a) (2^2)^3 = \underline{2^6}$$

$$b) 2^{(2^3)} = \underline{2^8}$$

$$c) ((-4)^2)^4 = \underline{4^8} = (2^2)^8 = \underline{2^{16}}$$

$$d) \left(\left(\frac{1}{3}\right)^3\right)^6 = \left(\frac{1}{3}\right)^{18} = \underline{\frac{1}{3^{18}}}$$

$$e) \left(-\frac{2^4}{3^3}\right)^2 = \frac{2^8}{3^6}$$

$$f) \left(\frac{2}{3}\right)^3 \div \left(\frac{5}{3}\right)^3 = \frac{2^3}{3^3} : \frac{5^3}{3^3} = \frac{2^3}{\cancel{3^3}} \cdot \frac{\cancel{3^3}}{5^3} = \frac{2^3}{5^3}$$

$$g) 4^2 \cdot 2^5 \cdot 8^2 = (2^2)^2 \cdot 2^5 \cdot (2^3)^2 = 2^4 \cdot 2^5 \cdot 2^6 = 2^{4+5+6} = 2^{15}$$

$$h) \left(\frac{3}{4}\right)^4 \div \left(\frac{9}{8}\right)^4 = \frac{3^4}{4^4} : \frac{9^4}{8^4} = \frac{3^4}{4^4} \cdot \frac{8^4}{9^4} = \left(\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}}\right)^4 = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$$

$$i) \frac{(3 \cdot 9 \cdot 27 \cdot 81)^5}{3^{50}} = \frac{(3 \cdot 3^2 \cdot 3^3 \cdot 3^4)^5}{3^{50}} = \frac{(3^{1+2+3+4})^5}{3^{50}} = \frac{3^{50}}{3^{50}} = 1$$

Ex 1.1.3

$$a) 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$b) 2^{-1} = \frac{1}{2}$$

$$c) 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

$$d) \left(\frac{1}{4}\right)^{-1} = 4^1 = 4$$

$$e) \left(\frac{-1}{2}\right)^{-2} = 2^2 = 4$$

$$f) \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

Ex 1.1.5

$$a) 2^4 \cdot 2^{-2} \cdot 2 = 2^{4+(-2)+1} = \underline{2^3}$$

$$b) (2^3)^{-5} = 2^{-15} = \frac{1}{\underline{2^{15}}}$$

$$c) \frac{5^3}{5^{-2}} = 5^{3-(-2)} = \underline{5^5}$$

$$d) ((-1)^{-2})^{-3} = (-1)^6 = \underline{1}$$

$$e) (2^{-1} \cdot 5^{-1})^{-1} = 2 \cdot 5 = \underline{10}$$

$$f) \left(\frac{11^{-2}}{11^8}\right)^{-5} = (11^{-10})^{-5} = \underline{11^{50}}$$

$$g) 7^{-3} \cdot \frac{49}{7^8} \cdot 7 = 7^{-3} \cdot \frac{7^2}{7^8} \cdot 7 = 7^{-3} \cdot 7^{-6} \cdot 7 = 7^{-3-6+1} = 7^{-8} = \underline{\frac{1}{7^8}}$$

$$h) 10'000 \cdot \frac{100}{100'000} \cdot 10^{-3} = 10^4 \cdot \frac{10^2}{10^5} \cdot 10^{-3} = 10^4 \cdot 10^{-3} \cdot 10^{-3} = 10^{-2} = \underline{\frac{1}{10^2}}$$

$$\left(i) \frac{1280 \cdot 5^7 \cdot 125}{(0,2 \cdot 25)^3} \stackrel{\otimes}{=} \frac{2^8 \cdot 5 \cdot 5^7 \cdot 5^3}{(5^{-1} \cdot 5^2)^3} = \frac{2^8 \cdot 5^{11}}{5^3} = 2^8 \cdot 5^8 = (2 \cdot 5)^8 = \underline{10^8} \right)$$

\otimes $1280 = 128 \cdot 10 = 2^7 \cdot 2 \cdot 5 = 2^8 \cdot 5$
 $0,2 = \frac{1}{5} = 5^{-1}$

Ex 1.1.6

$$a) x^2 y z^3 \cdot \underbrace{3xy}_{3^3} \cdot 27x^3 z^5 = 3^4 x^{2+1+3} y^{1+1} z^{3+5} = \underline{3^4 x^6 y^2 z^8} = \underline{81x^6 y^2 z^8}$$

$$b) (2a^2 b^3 c)^4 = \underline{2^4 a^8 b^{12} c^4} = \underline{16a^8 b^{12} c^4}$$

$$c) \left(\frac{2r^3}{s}\right)^2 \cdot \left(\frac{s}{r}\right)^3 = \frac{2^2 r^6}{s^2} \cdot \frac{s^3}{r^3} = \underline{2^2 r^3 s} = \underline{4r^3 s}$$

$$d) \frac{(4x^2 y^3)^5}{(2xy)^3} \div \frac{x^7}{(y^3)^4} = \frac{2^{10} x^{10} y^{15}}{2^3 x^3 y^3} \cdot \frac{y^{12}}{x^7} = \frac{2^{10} x^{10} y^{27}}{2^3 x^{10} y^3} = \underline{2^7 y^{24}}$$

$$e) (u^{-2} v^3)^{-3} = u^6 v^{-9} = \underline{\frac{u^6}{v^9}}$$

$$f) \frac{8x^3 y^{-5}}{4x^{-1} y^2} = 2x^{3-(-1)} y^{-5-2} = 2x^4 y^{-7} = \underline{\frac{2x^4}{y^7}}$$

$$g) \left(\frac{x}{3}\right)^{-2} \div \left(\frac{x}{9}\right)^{-3} = \left(\frac{3}{x}\right)^2 \div \left(\frac{9}{x}\right)^3 = \frac{3^2}{x^2} \cdot \frac{x^3}{9^3} = \frac{3^2 x}{(3^2)^3} = \frac{3^2 x}{3^6} = \frac{x}{3^4} = \underline{\frac{x}{81}}$$

$$h) \left(\frac{9y^3(3y^2)^{-2}}{(y^{-4})^{-3}}\right)^5 = \left(\frac{3^2 y^3 \cdot 3^{-2} y^{-4}}{y^{12}}\right)^5 = \left(3^0 y^{3+(-4)-12}\right)^5 = (y^{-13})^5 = y^{-65} = \underline{\frac{1}{y^{65}}}$$